Consequences of Mobile Ions on Plasma Wakefield Acceleration

Matthew M. Sieth
Department of Physics, University of Wisconsin Madison, Madison, WI 53706, USA

Warren B. Mori
Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, CA 90095, USA

In plasma wakefield accelerator (PWFA) schemes a very dense beam of electrons traverses a plasma, repelling plasma electrons from the axis. If the plasma ions remain stationary a uniform ion column results which provides a linear focusing force for both the driving and accelerated beams. Linear focusing is a cornerstone of PWFA schemes because it provides for ideal beam transport conditions. In our study we examine the effects of plasma ion motion due to beam fields. It has recently been suggested [1] that a collapse of the plasma ions towards the axis would destroy the linear focusing force making conventional PWFA schemes infeasible. We show that the affects of ion collapse might be minimized through careful selection of the beam and plasma parameters.

INTRODUCTION

Particle accelerators allow physicists to discover the nature of the fundamental forces, the structure of matter, and the beginning of the universe. There are also many practical applications for particle accelerators in different areas of industry and medicine. Currently there are a number of different accelerators both in operation and under construction that use RF waves to accelerate particles to relativistic energies. For example, the LHC (Large Hadron Collider) at CERN promises to discover the Higgs Boson in 2007 by colliding two proton beams at 14 TeV. Furthermore, plans for the ILC (International Linear Collider) at CERN have recently begun, which would be an electron-positron collider at ~ 1 TeV in the center of mass. The ILC may be able to answer questions about dark matter and extra dimensions that will be out of range for the LHC. As we strive to accelerate particles to even higher energies current RF technology begins to impose practical limitations. RF accelerators have achieved electric fields on the order of 10-20 MV/m. However, the electric fields cannot be made much larger because the accelerator structure will begin to be ionized. Therefore, in order to probe higher energy ranges particle accelerators must be made longer. Specifically, at 10-20 MV/m an accelerator would need to be on the order of 25 km long to accelerate an electron or positron beam to the 0.5 TeV range. In practice, the LHC is 27 km in circumference and the ILC will consist of two 20 km linear accelerators. The limitations that RF accelerators impose on the magnitude of the electric field are what bring us to consider the creation of a plasma accelerator. Since plasma is already in an ionized state, the issue of unwanted ionization in a plasma accelerator would not be a problem. Plasma accelerators offer the potential for acceleration gradients on the order of 10-20 GeV/m for electron or positron beams, a one-thousand fold increase over conventional technology.

For this reason, the future of high energy physics may largely reside in the realm of plasma acceleration.

PLASMA ACCELERATORS

Plasmas contain ions and electrons and are generally electrically neutral as a whole. However, disturbances in the plasma can create local regions of positive or negative charge concentration. The electrons and ions will then move in such a way that the neutrality of the plasma is maintained. In the case of a time-varying disturbance it is possible to create a plasma wave in which the charge density of the plasma varies sinusoidally along the longitudinal direction of the accelerator. It is important to note that since ions are much more massive than electrons, the ions will remain essentially stationary and so it is the electrons that oscillate about the axis. Finally, once a plasma wave is established there is an associated longitudinal electric field which is capable of accelerating beams of electrons to relativistic speeds. There are four different plasma accelerator schemes: (1) the plasma beat-wave accelerator, (2) the plasma wake-field accelerator (PWFA), (3) the laser wake-field accelerator (LWFA), and (4) the self-modulated wake-field accelerator (SMLWFA). All of these schemes are similar in that they involve either a particle beam or laser driver that traverses the plasma and thereby creates a plasma wave.

The beat-wave method predates all of the other plasma accelerator schemes and for this reason has been studied more in both theory and experiment. This scheme utilizes two lasers which are at slightly different frequencies such that the difference in frequency equals the plasma frequency (i.e. ω_2-ω_1=ω_p). The superposition of these two lasers will result in "beat-waves" that occur at the plasma frequency. As the beat-wave propagates its radiation pressure resonantly excites the plasma which results in a longitudinal plasma wave. The reason that so much research has been done in past decades on the beat-wave method instead of the other acceleration schemes lies in the fact that it doesn't require ultra-short laser pulses which are required for the other laser methods. Only recently have short pulse lasers become readily available.

The wake-field methods for acceleration utilize either an electron beam or a laser pulse which traverses the plasma and expels electrons outward which ultimately results in a plasma wave. The process is exactly analogous to a boat traveling through water. A boat pushes water outward, which causes a wake to form behind the boat where the water from both sides of the boat rushes inward and joins together. Likewise, the electron beam expels plasma electrons via Coulomb repulsion, while the laser pulse does so through radiation pressure. The electron blowout region can be seen in figure (2) which shows the electron density in a PWFA computer simulation. The corresponding electron beam is shown in figure (1). After the electron beam or laser pulse passes a section of plasma the blown out electrons in this region rush back towards the axis to reestablish charge neutrality. However, when the electrons reach the axis they have a finite amount of kinetic energy which causes them to overshoot. The electrons then continue to

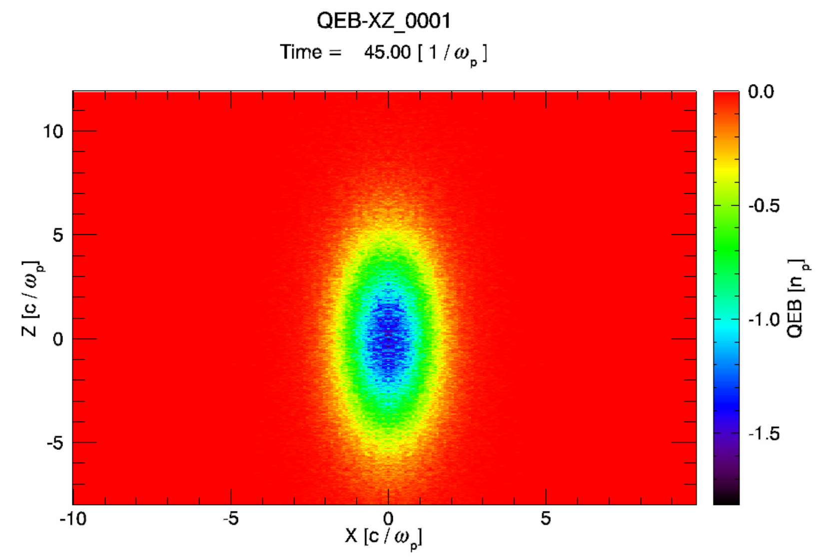

Figure 1. Charge density plot of the electron beam. The beam in this simulation is tri-Gaussian.

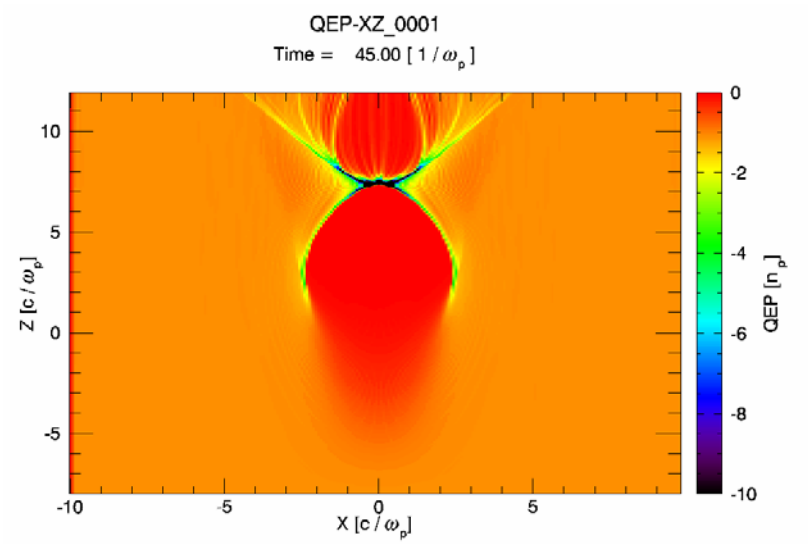

Figure 2. Charge density of the plasma electrons. The plasma electrons are blown out by the electron beam, leaving a uniform ion column along the axis.

oscillate about the axis due to electron-ion attraction at which point a plasma wave is established with a phase velocity equal to the speed of the driver (around c).
Once a plasma wave is established through one of these schemes there exists an associated longitudinal electric field. If a trailing beam of electrons is put at a location behind the driving beam where there is a strong negative electric field the beam will be accelerated, at which point we have a plasma accelerator.

After the electron expulsion in a plasma accelerator, a concentration of positive charge exists along the axis where only ions remain and is referred to as the ion column. This ion column has a uniform charge density – assuming that the ions remain mobile due to their large inertia - and is cylindrically symmetric and so by Gauss's Law we know that the force is proportional to the distance from the axis. It turns out that this linear focusing force is critical in preserving the quality in both the driving and accelerating beam a topic which is discussed more in a later section.

AFTERBURNER CONCEPT

Much focus has been put on the PWFA because of its potential to be used as an "afterburner" to an RF linear collider [2]. In the afterburner concept, a PWFA around 30 meters long would be placed at the collision point of an electron-positron collider. The electron and positron beams would then be split into two microbunches before entering the afterburner where the first bunch would serve as the driving beam and the second would function as the accelerated beam. A schematic of an afterburner is shown in figure (4). Using this method the energy of the linear collider could potentially be doubled (or more). With this said, the afterburner concept looks to be a very feasible way of extending the energy range of conventional RF accelerators.

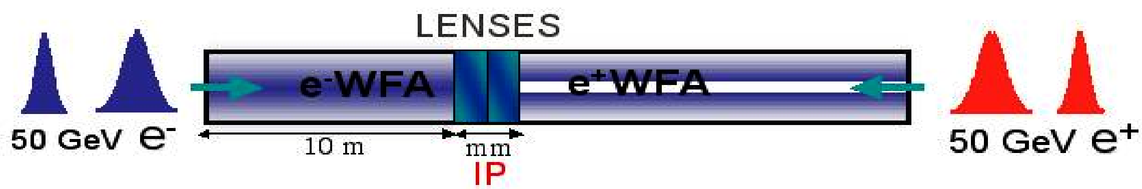

Figure 3. Schematic of the plasma afterburner.

QUICKPIC

Plasma accelerator simulations are commonly carried out by using particle in cell (PIC) methods. In the fully explicit PIC algorithm plasma particles reside inside a gridded simulation box. The charge and current densities of the particles are then deposited to the nearest grid points. Using these charge and current densities the electric and magnetic fields can then be calculated via Maxwell's equations:

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

At this point the force on the particles is determined by the relativistic Lorentz force, $\frac{d\vec{p}}{dt} = q\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right)$. The positions are then advanced according to this force at which point the fields are recalculated and the entire process repeats.

The fully explicit PIC algorithm is extremely useful and has given us tremendous insight on plasma accelerators, but is also very computationally intensive due to its brute force nature. To run our simulations we used a newly developed software called QUICKPIC [3] which makes use of some timesaving approximations. QUICKPIC is a fully three-dimensional code that is able to model both PWFA and LWFA. It utilizes what is referred to as the quasi-static approximation (QSA). In the QSA it is assumed that the driver evolves on a much slower time scale than the plasma particles. Therefore, the beam can essentially be propagated forward 100's to 1000's of plasma particle time steps before being updated.

The QUICKPIC code makes a mathematical transform from the Cartesian coordinates (x, y, z, t) to the speed of light coordinates (x, y, s, ξ), where s=z and ξ =ct-z. The variables s and ξ both have units of length but correspond to different time scales; the s coordinate corresponds to the slow timescale over which the driving beam develops while ξ corresponds to the fast time scale and relates to the evolution of the plasma particles. Invoking the QSA mathematically means that $\partial s < \partial \xi$.

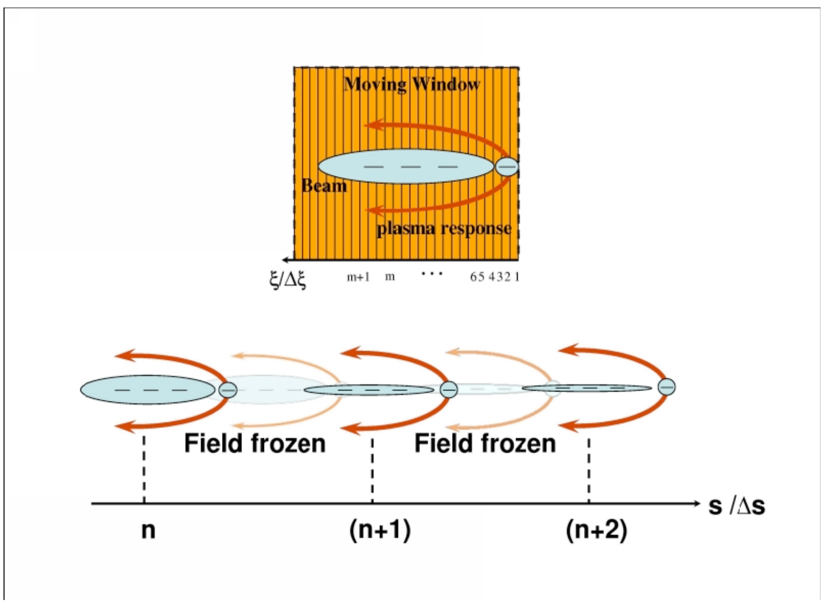

Figure 4. Illustration of the speed of light variables *s* and *ξ*. The driving beam is updated when *s* is advanced while the trajectories of the plasma electrons are tracked in *ξ*.

The QUICKPIC scheme begins with Maxwell's equations in the Lorentz Gauge:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)\phi(x,y,z,t)=4\pi\rho(x,y,z,t) \tag{1}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)\vec{A}(x,y,z,t)=\frac{4\pi}{c}\vec{J}(x,y,z,t) \tag{2}$$

Using the fact that $\frac{\partial}{\partial z} = \frac{\partial}{\partial s} - \frac{\partial}{\partial \xi}$ and $\frac{\partial}{\partial t} = c\frac{\partial}{\partial \xi}$ and the assumption that $\partial s < \partial \xi$, equations (1) and (2) can be rewritten as:

$$\nabla_{\perp}^2 \phi(x, y, s, \xi) = -4\pi\rho(x, y, s, \xi) \tag{3}$$

$$\nabla_{\perp}^2 \vec{A}(x, y, s, \xi) = -\frac{4\pi}{c}\vec{J}(x, y, s, \xi) \tag{4}$$

Invoking the fact that the beam moves in the *z* direction we have:

$$\nabla_{\perp}^2 \phi(x, y, s, \xi) = -4\pi\left(\rho_e(x, y, s, \xi) + \rho_b(x, y, s, \xi)\right) \tag{5}$$

$$\begin{aligned}\nabla_{\perp}^2 A_z(x, y, s, \xi) &= -\frac{4\pi}{c}\left(J_{z,e}(x, y, s, \xi) + J_{z,b}(x, y, s, \xi)\right)\\ &= -\frac{4\pi}{c}\left(J_{z,e}(x, y, s, \xi) + c\rho_b(x, y, s, \xi)\right)\end{aligned} \tag{6}$$

Defining $\psi = \phi - A_z$ the difference of (5) and (6) gives us:

$$\nabla_{\perp}^2 \psi(x, y, s, \xi) = -4\pi\left(\rho_e - \frac{J_{z,e}}{c}\right) \tag{7}$$

In this way, the problem of solving for the potentials has been reduced from a fully three-dimensional problem to a two-dimensional Poisson equation. Once the potentials have been determined for every transverse section of plasma the electric and magnetic fields directly follow. In particular the longitudinal electric field, E_z, can be determined through the relation:

$$E_z = \frac{\partial \psi}{\partial \xi} \tag{8}$$

Figure (5) shows a plot of E_z as a function of *z*, which was obtained via equation (8). It is clear from this plot that the beam loading position is around z=5 c/ω_p where the electric field is large and negative.

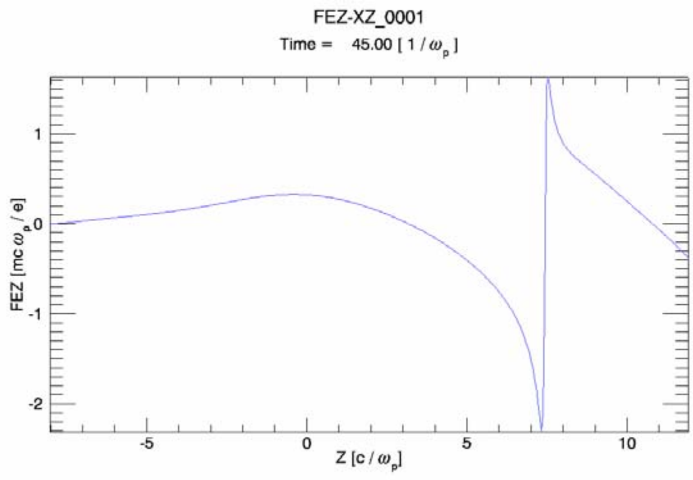

Figure 5. Longitudinal electric field for a QUICKPIC PWFA simulation.

The equations of motion are distinct for the beam particles and plasma particles. For beam particles the equations of motion are integrated with respect to s, which takes the role of a time-like variable for the fast time scale:

$$\frac{d\vec{p}_b}{d(s/c)} = q_b\left(\vec{E} + \frac{1}{c}\vec{v}_b \times \vec{B}\right) \tag{9}$$

On the other hand the equations of motion for plasma particles are integrated with respect to ξ, the time-like variable in the slow time scale:

$$\frac{d\vec{p}_e}{d(\xi/c)} = \frac{q_e}{1-\vec{v}_{ez}/c}\left(\vec{E} + \frac{1}{c}\vec{v}_e \times \vec{B}\right) \tag{10}$$

Once the forces are solved for the particles are then advanced at which point the potentials are found once again (from which the fields directly follow) and the force is solved for again.

As a final note we must address how the beam evolution and plasma evolution are integrated. While the process of solving for the fields and pushing the plasma particles is entirely two-dimensional the evolution of the beam is three-dimensional. Therefore QUICKPIC requires a parallel processing scheme where the two-dimensional routine is embedded inside the three-dimensional routine.

FOCUSING FORCE

In the plasma wakefield accelerator the driving beam repels plasma electrons creating an electron-rarefied region also known as the blowout regime. In this blowout regime only the ions - which are considered to be stationary - remain. Therefore the charge density in this region is completely homogeneous. Considering this uniform charge density, we know from Gauss's Law that the force on a particle inside this region will be linear in *r*. Therefore the electrons in the driving and accelerated beams – both of which are located within the blowout regime – will experience simple harmonic motion about the axis. We then wish to show that this linear focusing force, which results in simple harmonic motion for individual beam particles, will lead to ideal transport characteristics for the beams.

We start with a distribution of particles in phase space, $f(\vec{x},\vec{v};t)$. Conservation of particles leads to the Vlasov equation:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f + \vec{a}\cdot\vec{\nabla}_v f = 0 \tag{11}$$

which is valid for the PWFA scheme because the phase velocity of the wakefield is much larger than the thermal velocity of the plasma. The operator D/Dt is known as a convective derivative. The fact that $Df/Dt = 0$ is equivalent to saying that *f* does not change in a frame of reference that moves with the particles in phase space.

Jean's Theorem then states that the Vlasov equation is satisfied for any function $f(\alpha_j(\vec{x},\vec{v},t))$ that is dependent on any constants of the motion. In our simulations we used a beam which initially had a Gaussian spatial and velocity distribution in the transverse direction (which *does* satisfy the Vlasov equation):

$$f_0 = c_1 e^{-\alpha_0 x_0^2} e^{-\beta_0 v_0^2} \tag{12}$$

where $\alpha_0 = \dfrac{1}{2\sigma_x^2}$ and $\beta_0 = \dfrac{1}{2v_{thermal}^2}$.

We first consider the case where the particles are free-streaming as time evolves, meaning that $x = x_0 + vt$ and $v = v_0$. In the case of free-streaming particles, it is a special property of a Gaussian distribution that the phase space distribution will remain Gaussian in time. However, as the beam propagates its spot size, σ_r, will increase according to the expression:

$$\sigma_r(z) = \sigma(0)\sqrt{1+\frac{c^2t^2}{\beta^{*2}}} \tag{13}$$

Accordingly, β^* is defined as the distance the beam must travel before its spot size increases by a factor of $\sqrt{2}$. For the distribution given in equation (12) it can be shown that $\beta^* = \dfrac{1}{c}\sqrt{\dfrac{\beta_0}{\alpha_0}}$.

We then consider the case where a linear focusing force acts on the electrons due to the uniform ion column. In this case the particles will undergo simple harmonic motion:

$$x(t) = A\cos(\omega t + \phi_0) \tag{14}$$
$$v(t) = -\omega A\sin(\omega t + \phi_0) \tag{15}$$

Solving these equations for x_0 and v_0 we find that:

$$x_0 = x(0) = x(t)\cos(\omega t) - \frac{v(t)}{\omega}\sin(\omega t) \tag{16}$$
$$v_0 = v(0) = v(t)\cos(\omega t) + \omega x(t)\sin(\omega t) \tag{17}$$

Substituting x_0 and v_0 into f_0, we retrieve the phase space density as a function of time,

$$f(\vec{x},\vec{v};t) = c_1 e^{-\alpha_0\left(x(t)\cos(\omega t) - \frac{v(t)}{\omega}\sin(\omega t)\right)^2} e^{-\beta_0(v(t)\cos(\omega t)+\omega x(t)\sin(\omega t))^2} \tag{18}$$

which can also be written in the form:

$$f(\vec{x},\vec{v};t) = c_1 e^{-\alpha(x(t))^2} e^{-\beta(v(t)-c_2)^2} \tag{19}$$

where $\alpha = \dfrac{\alpha_0}{\dfrac{\sin^2(\omega t)}{\beta^{*2}c^2\omega^2} + \cos^2(\omega t)}$

and $\beta = \alpha_0\left(\dfrac{\sin^2(\omega t)}{\omega^2} + \beta^{*2}c^2\cos^2(\omega t)\right)$

Integrating Equation (19) in v then gives us the spatial beam distribution $f(\vec{x};t)$

$$f(\vec{x};t) = \int f(\vec{x},\vec{v},t)dv = c_2 e^{-\alpha(x(t))^2} \tag{20}$$

We can then solve for the spot size σ_r:

$$\sigma_r = \frac{1}{\sqrt{2\alpha}} = \left(\frac{1}{2\beta_0}\frac{\sin^2(\omega t)}{\omega^2} + \frac{\cos^2(\omega t)}{2\alpha_0}\right) \tag{21}$$

It can then be verified that σ_r satisfies the wave envelope equation:

$$\ddot{\sigma}_r + \sigma_r\left(k_\beta^2 - \frac{\varepsilon_N^2}{\gamma^2\sigma_r^4}\right) = 0 \quad (22)$$

The beam is considered to be matched if $k_\beta^2 = \frac{\varepsilon_N^2}{\gamma^2\sigma_r^4}$ so that $\ddot{\sigma}_r = 0$. Therefore if the beam is initially focused (i.e. $\dot{\sigma}_r = 0$) the spot size will remain constant in time. However, if the beam is not matched it will experience betatron oscillations where the spot size of the beam oscillates with a frequency that is half of the oscillation frequency of the individual electrons.

Linear focusing is also important in controlling the emittance of the beam. The emittance is a measure of beam quality and is given approximately by $\varepsilon \sim \pi\sigma_r v_{th}$. We can show through a simple, but not rigorous argument that linear focusing (or simple harmonic motion) leads to a constant emittance. We first imagine a Gaussian distribution of particles in phase space. Simple harmonic motion will then cause the beam particles to travel in a circular trajectory in phase space about the origin. A matched beam then corresponds to a distribution of particles where the standard deviation of the spatial and velocity distributions are equal. As time elapses the particles will continue to move in a circle but the spot size and thermal velocity will remain constant. Therefore the emittance is also constant. Note that it is not necessary for the beam to be matched in order for the emittance to be preserved. Linear focusing of an unmatched beam will also result in constant emittance.

From this discussion it is clear that a linear focusing force is a cornerstone of the PWFA method. Linear focusing allows for a beam with a constant spot size and high quality, which is critical in transporting the beams in this scheme.

MOBILE IONS

In most PIC simulations it is approximated that the plasma ions remain completely stationary in the PWFA schemes. In reality, the plasma ions do move as a result of the beam fields, but this motion is typically assumed to be very small due to the inertia of the ions. However, [Rozenzweig et. Al, 2005] proposed that there can be a complete collapse of the ions towards the axis in the PWFA scheme. Such a collapse would destroy the uniform density of the ion column and which would result in a focusing force nonlinear in *r*. The emittance of the beam would then grow and the spotsize of the beam would then vary in time according to equation (22) which could result in emittance growth.

For their simulations [Rozenzweig et. Al, 2005] used the parameters shown in Table 1.

Table 1. Paramers used in the PIC simulation by Rosenzweig, et al.

Ions	Hydrogen
n_p	$2 \cdot 10^{16}$ cm^{-3}
N_b	$3 \cdot 10^{10}$
σ_z	63 μm
σ_r	172 nm

Using these simulation parameters in QUICKPIC we obtained a result consistent with the Rosenzweig study. A plot of the plasma ion density is shown in figure (5).

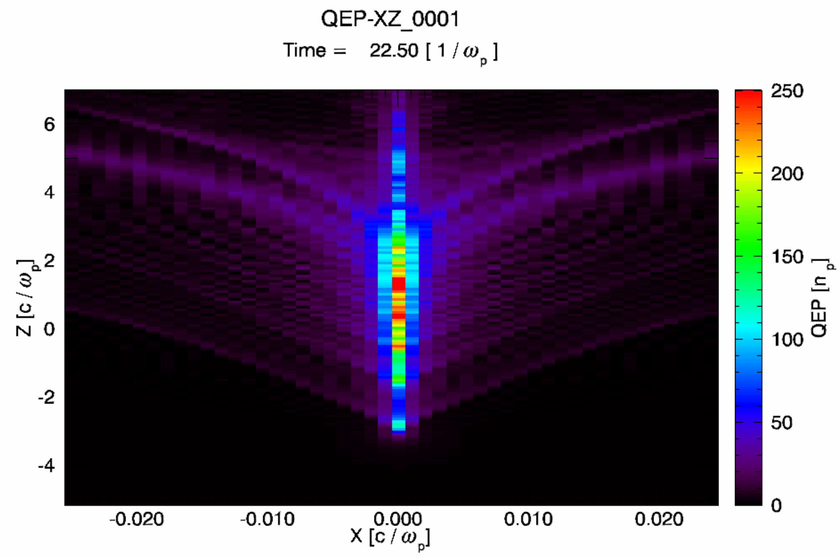

Figure 6. Plot of the plasma charge density for the parameters given in Table (1).

As seen in this plot the plasma ions collapse completely on axis reaching charge densities several thousand times greater than the background ion density. As expected the focusing force becomes extremely nonlinear.

LARGER SPOT SIZE

One way to decrease the effect of ion collapse is to reduce force seen by ions near the axis. To do this we must reduce the peak charge density of the drive beam. The charge density n_b is given by the expression:

$$n_{b,0} = \frac{N_b}{(2\pi)^{3/2}\sigma_r^2\sigma_z} \tag{23}$$

A simple way to decrease the beam density is to increase σ_r. Figure (7) shows the average ion density within a circular cross-section (of radius *r*) of the plasma as a function of *r*. The cross section is taken from the longitudinal position which corresponds to the position of the trailing beam (ξ=5 c/ω_p) .

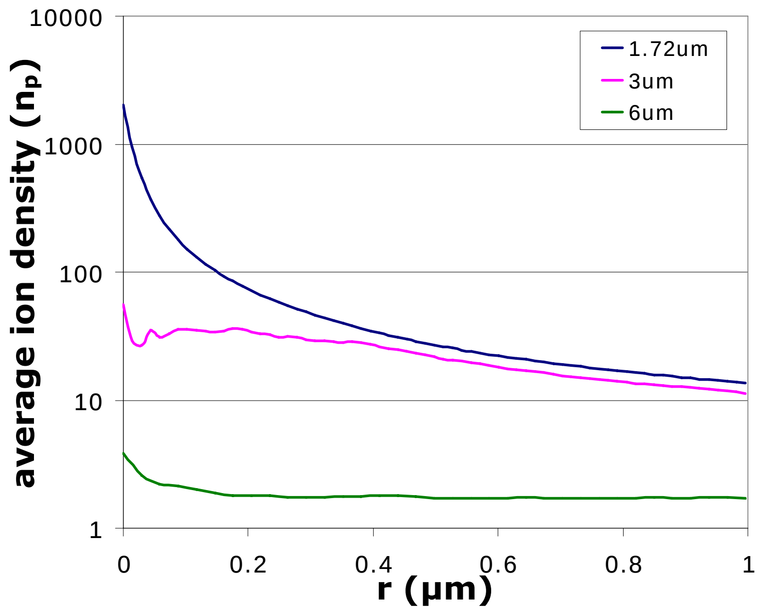

Figure 7. Average ion density of a cross sectional area of plasma as a function of *r* in µms for different spot sizes.

Since the spot size of the trailing beam would be submicron figure (7) only includes *r* values up to a micron despite the fact that the simulation box was considerably larger. Clearly an increase in spot size decreases the amount of charge that builds up on axis. More importantly though is the effect that this ion build up has on the focusing force within the blowout regime. Figure (8) shows the focusing force for these simulations.

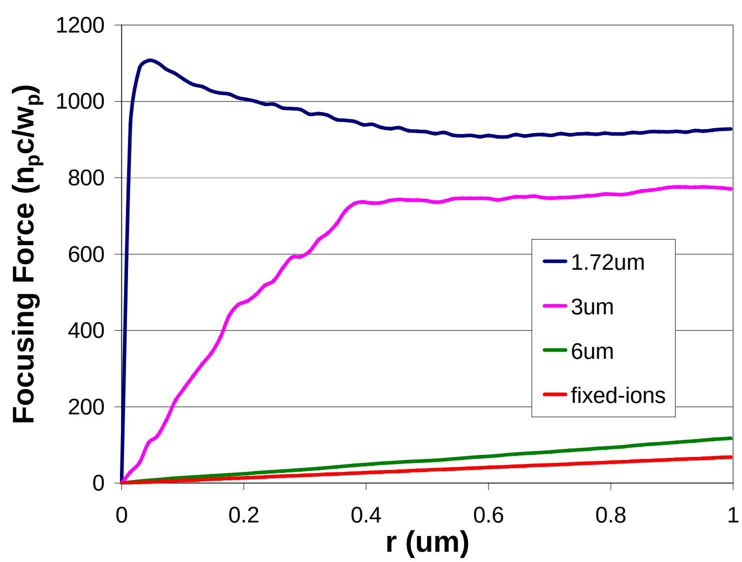

Figure 8. Focusing force exerted on the trailing beam due to the ion column.

For the 1.72 μm and 3 μm spot size simulations the focusing force is very nonlinear, which is undesirable for the PWFA. However, the 6 μm spot size simulation has a nearly linear focusing force which indicates that using a large enough spot size may allow for reasonable transport of the trailing beam.
The issue with using larger spot sizes for the drive beam is the radiation losses due to the betatron motion.

HEAVIER IONS

Another possible solution to the ion collapse problem is to use heavier ions. The more massive ions will see less acceleration and will therefore remain more nearly stationary. However, it is not possible to use arbitrarily large ions because of practical limitations. In most PWFA schemes the plasma inside the accelerator is created through a self-ionization method. As the driving beam traverses atoms in a vapor form the beam fields ionize these atoms resulting in plasma. Therefore the atoms used in the accelerator must have a first ionization energy that is sufficiently small so that the self-ionization can take place while the second ionization energy must be sufficiently large so that the plasma contains only one type of ion. Both Hydrogen and Lithium exhibit these ionization properties. Therefore we chose to repeat the simulations from the previous section with Lithium ions (M=7) instead of Hydrogen ions (M=1). Lithium is also a reasonable choice for practical reasons since it is Lithium that is used in the PWFA experiments being run at SLAC.

Figure (9) shows the average ion density as a function of r for a two-dimensional cross section at the beam-loading location with Lithium ions. Comparing figures (7) and (9) we see that increasing the mass of the ions seven-fold dramatically reduces the amount of charge that builds up on axis. Figure (10) shows the focusing force for these simulations that were run with Lithium. In this case all of the focusing forces are much more nearly linear. In fact, for the 3 μm and 6 μm spot sizes the focusing force is greater than that of a PWFA where the ions are fixed, but nonetheless look completely linear. We propose that it may be possible to utilize this characteristic in order to construct a plasma lens. The focusing force that is created with mobile ions is greater than that of the fixed ion scenario. Therefore, if the focusing force remains linear in the submicron range this increased focusing force could be used to focus the trailing beam in the final stages of a PWFA without compromising beam quality. This concept requires further study and will be investigated in future research.

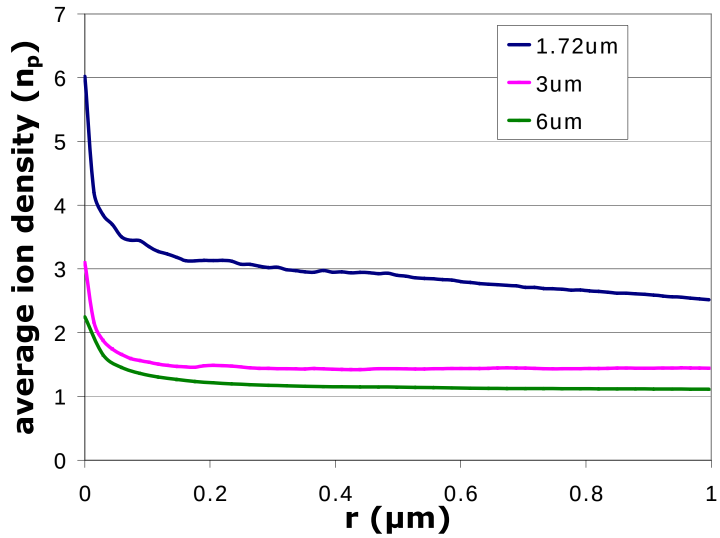

Figure 9. Average ion density of a cross sectional area of plasma as a function of *r* for different spot sizes. Lithium ions are used. Compare to Figure 8 which uses Hydrogen ions.

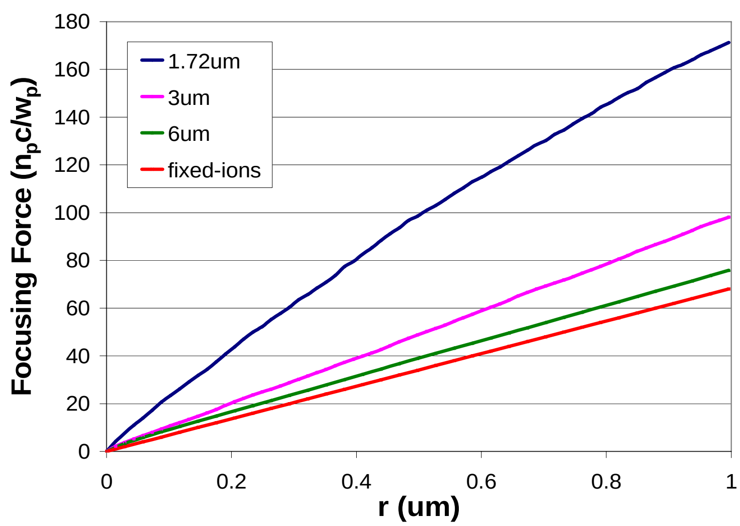

Figure 10. Focusing force exerted on the trailing beam due to the ion column. Lithium ions were used in this simulation.

CONCLUSION

The issue of plasma ions collapsing towards the axis due to the driving beam fields poses a serious concern for the PWFA scheme. In our study we confirmed the basic results of Rosenzweig et. al, showing that extreme ion collapse can occur with certain beam and plasma parameters. We also showed that the ion collapse problem might be circumvented via two routes: (1) A greater beam spot-size can be used. By increasing the spot size the peak beam density becomes smaller and so the ions feel less force. (2) Heavier ions can be used so that the ion motion is minimized. By changing from

Hydrogen ions to Lithium ions – which are typically used in experiment – we demonstrated in our simulations that ion collapse is greatly reduced. Finally, we note that the ion collapse phenomenon may be beneficial for use as a plasma lens. However, further research must be done to test this idea.

ACKNOWLEDGMENTS

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