Computer simulations of ion motion in beam driven plasma wakefield accelerators

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Abstract

Issues related to plasma based acceleration using nonlinear wakefields are investigated. Nonlinear wakefields provide ideal linear focusing fields and radially independent accelerating fields which limit emittance growth and energy spread. When the accelerated charge is tightly focused the plasma ions can collapse thereby modifying the wakefields within the beam. We use computer simulations to characterize the severity of ion collapse in an intense beam driven PWFA and the attendant deviation from linearity. In particular, the moderating effects of ion mass and plasma temperature were evaluated; the former had a significant effect and the latter appeared insignificant for typical cases. Insufficient resolution prohibits quantitative description of these effects, but they scale predictably, and so the results are qualitatively accurate.

I. Motivation for plasma wakefield acceleration

High energy particle accelerators are the fundamental tool used to discover new elementary particles. The cost of the accelerators at the energy frontier have already reached a limit that makes the construction of a high energy accelerator doubtful unless a new technology is developed. The cost of the recently completed Large Hadron Collider and the projected cost of the proposed International Linear Collider makes this painfully clear.

Plasma wakefield accelerators (PWFAs) have recently attracted examination as an economical and more efficient replacement for conventional radio frequency (RF) accelerators. For example, whereas the upper limit on an acceleration gradient for RF accelerators is on the order of tens of megavolts per meter, acceleration gradients greater than that by a factor of 1000 have already been demonstrated in PWFAs for lengths up to 1 meter. Furthermore, plasma based acceleration mechanisms could provide cheap tabletop accelerators for medical and industrial uses.

Laser driven PWFAs were first proposed in the 1970s by the late John Dawson of UCLA; a student of his proposed in the 1980s to instead shape the plasma with a dense positron or electron beam instead.

In addition to simulations, PWFAs have been successfully tested in concept. In an experiment at the Stanford Linear Accelerator Center the concept of a plasma afterburner was studied. A uniform column of gas was ionized by the electric field of an intense driving beam with an initial energy of 42-GeV. Electrons at the head of the beam create a wakefield on which electrons in the tail of the beam surf to an energy of 85 GeV in only .85m.

II. Relevant theory

The complicated and precise process of PWFA is easy to imagine and to describe qualitatively. Some of the approximations involved are more subtle, yet they are required for full understanding.

Two beams of electrons are involved in the acceleration. A driving beam creates a plasma wakefield on which a trailing beam surfs to high energy. As it creates the wakefield the driving beam loses energy, while trailing beam absorbs the wake as it gains energy. In effect, the trailing beam is accelerated at the expense of the driving beam.

To start the process, the driver, a dense (n_b=10^4 n_p) relativistic electron beam, is directed into a neutral cold plasma. Because the beam has very high energy (tens of GeV), the effective mass of each particle in it is high enough to render it resistant to changes in its forward path. Likewise, the heavy ions of the plasma do not move much during the time it takes the driver to pass by it because of their high mass.

The high charge density of the beam strongly repels the plasma electrons; they move away from the beam axis, and as the beam continues forward, the electrons coalesce into a sheath around a trailing ion column that is completely vacated of electrons; since the ions nearly do not move, inside the ion column is a nearly uniform distribution of positive charge. The electron sheath collapses back toward the axis within a plasma period behind the driving beam; incidentally, this length (the speed of light times the plasma period) is on the scale of micrometers, which presents difficulty in placing the trailing beam. The plasma electrons then overshoot and oscillate about the axis for several more wavelengths, but from an argument to follow, this has little effect on beams of interest to us.

The longitudinal electric field in the ion column is, predictably, positive in the front (slowing down the driver) and negative in the back (accelerating the trailing beam). The focusing (transverse) force, due to electric and magnetic fields for electrons moving forward at the speed of light, is nearly perfectly linear. This will be characterized more precisely in the following paragraphs.

A. Self consistent beam coherence

No bunch of electrons will stay stationary if it's very dense, due to Coulomb repulsion. However, a moving beam also contains a current. The azimuthal magnetic field generated by a charge moving very close to the speed of light perfectly balances out the radial electric field. Therefore if a charged particle beam is moving very close to the speed of light, it expands very slowly due to its own fields.

B. Quasi-static approximation

Maxwell's equations in the Lorentz gauge are

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\phi = -\frac{1}{\epsilon_0}\rho$$

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\vec{A} = -\mu_0\vec{J}$$

We can perform a change of coordinates

$$(x, y, z, t) \to (x, y, s, \xi)$$

where x and y are unchanged and

$$s = z$$

$$\xi = ct - z$$

This transformation can be thought of as introducing a moving box inertial reference frame, moving at c. The location of the origin of the box is indexed by *s*, and the position within the box is indexed by ξ. This is sufficient to recover the initial *z* coordinate if needed. The box moves with the speed of the electron beams (≈c), so the transformation has the effect of fixing one of the coordinates for the beam particles, i.e. ξ.

The expressions for partial derivatives then become

$$\partial_z = \partial_s - \partial_\xi$$

$$\partial_t = c\partial_\xi$$

At this point, a timely approximation greatly simplifies the following computations. Since the beam is highly relativistic (hence very "stiff"), it changes very little in response to the new plasma it encounters. For this reason, we can infer that even as the beam travels over several plasma wavelengths, the effect it has on the plasma will be the same. That is, within the moving box, the plasma surrounding the beam will retain the same shape until the beam has had a chance to appreciable evolve. The relevant timescale for beam evolution is that of betatron oscillation

$$\omega_\beta = \frac{\omega_p}{\sqrt{2\gamma}}$$

which is at least a hundred times slower than plasma oscillation. This allows us to make the *quasi-static approximation*

$$\partial_s \ll \partial_\xi$$

which tells us that the partial derivative for any relevant plasma or beam quantity (in particular the displacement of a particle) changes much more slowly with *s* than with ξ. Therefore

$$\partial_z \approx -\partial_\xi$$

$$\partial_t = c\partial_\xi$$

Substituting these partial derivatives into the Maxwell equations, we get a significant simplification:

$$\nabla_\perp^2 \vec{A} = -\mu_0 \vec{J}$$

$$\nabla_\perp^2 \phi = -\frac{1}{\epsilon_0}\rho$$

where the gradient is taken only with respect to the perpendicular coordinates *x, y*. This reduces the problem to solving Poisson equations in two dimensions. A significant consequence of this is that all of the electric and magnetic fields in a perpendicular (x,y) slice follow from two-dimensional electrostatics and magnetostatics for the charge and current distributions in that slice.

C. Emittance preservation

There are several measures of beam quality. A beam should ideally be narrow in spatial spread, narrow in energy and narrow in thermal velocity spread. *Emittance* is the hypervolume that a particle distribution occupies in phase space. Therefore, the smaller the emittance the better. It is not straightforward to reduce the emittance (cool) a beam. Beams with smaller emittances can be focused to smaller spot sizes.

Gaussian beams were exclusively used for the simulations. Under a linear focusing force, each particle in the beam undergoes simple harmonic motion (SHM) with the same frequency. Each period of SHM brings the particles back to their initial positions and velocities. For an ensemble of electrons the spot size will oscillate and for a linear force if the beam has an initial Gaussian profile then it will reman Guassian with a time dependent spot size. It is also possible to create a “matched beam” for which the spot size does not oscillate and the emittance is still preserved. For these cases there is a balance between the thermal pressure and the focusing forces which are causing the betatron oscillations. The condition for a matched beam can be written as

$$\sigma_{v_{th}} = \omega_\beta \sigma_{x0}$$

If the focusing force is nonlinear then different parts of the beam oscillate at different frequencies and the phase mixing can lead to emittance growth. However, it is possible to find a matched transverse profile that deviates from a Gaussian for a given nonlinear force.

Using the quasi-static equations for the potentials it is simple to show that the electric field due to a cylindrical uniform charge distribution increases that varies along z-ct increases linearly away from the axis. However, as suggested in the preceding sections, the ions do not remain completely stationary and hence are not uniform because they move inward toward the beam axis, attracted by the dense negative charge of the beam.

The resulting deviation from linear focusing is the subject of the following simulations.

III.Computational techniques

It is computationally infeasible to calculate exactly the electromagnetic fields that arise from a distribution of millions or billions of particles. For this reason, the plasma simulation group at UCLA uses Particle In Cell (PIC) simulations, which make the computational problem tractable but yet retain the capacity for capturing all the nonlinearities and resolution given sufficient processing capacity.

The PIC code starts with an actual set of particles, with their respective associated positions and momenta. Next, the charges are deposited onto a discrete grid of allowed position values, placing a part of the charge onto each grid point proportional to its proximity to that point. Currents are found from the charge and the momentum.

The program then calculates the electric and magnetic fields at each grid point due to all the other grid points. From there, field values are interpolated at the locations of the particles which lie in between the grid points. These fields are used to calculate the momentum change due to the Lorentz (electromagnetic) force during a time step. The particles are then pushed to new positions based on their velocity. The process is then repeated.

A "full" PIC code, such as Osiris, does this for many particles and with time steps that resolve the shortest time scales of interest. However, some approximations to this process can greatly lower computational demands. The code I used was QuickPIC, a PIC code that takes advantage of the quasi-static approximation.

Because the beam is stiff and evolves much more slowly than the surrounding plasma, QuickPIC takes the beam as stationary. In the quasi-static approximation, fields depend only on charge distributions only in their own slice. Taking advantage of that, QuickPIC takes a slice of new neutral plasma and moves it back in ξ, keeping track of how it looks at each ξ. This is the computationally intensive 2D step.

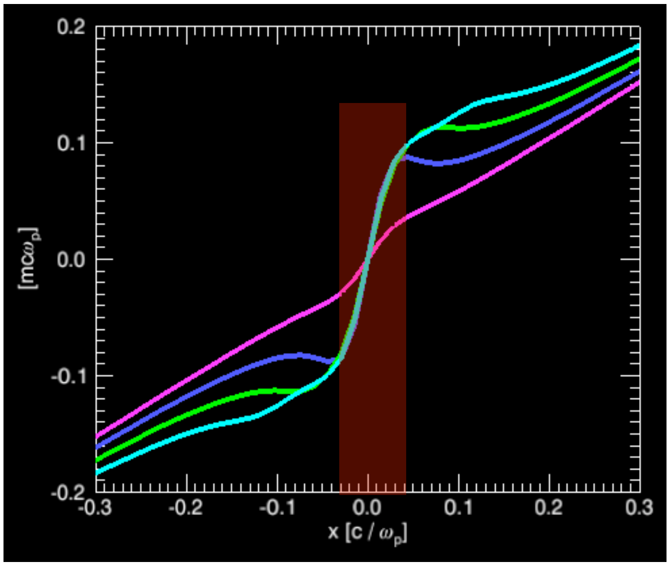
There are significant computational cost savings from using QuickPIC, because the fundamental calculation is in two rather than three dimensions.

IV.Simulations undertaken

I ran various QuickPIC simulations, most often on UCLA's Dawson cluster, a cluster of 256 dual processor 2.0/2.3 GHz Xserve nodes; and occasionally on NERSC/LBNL's Franklin cluster, which has 9660 dual core 2.6 GHz nodes.

The parameters that were kept constant across all the simulations were as follows: hydrogen plasma with $n_p = 10^{17}cm^{-3}$; $c/\omega_p = 16.8\mu m$; $n_b=10^4 n_p$; $\sigma_x=\sigma_y=0.328\mu m$; $\sigma_z=10\mu m$; $\gamma=5\times10^5$ (250 GeV). This beam is much wider than a typical beam in an accelerator setting, but the resolution of my simulations was too low, which necessitated that I use a wider beam containing correspondingly more charge, so that the peak density remained the same.

Figure 1. Focusing (transverse) force at different places within the beam.



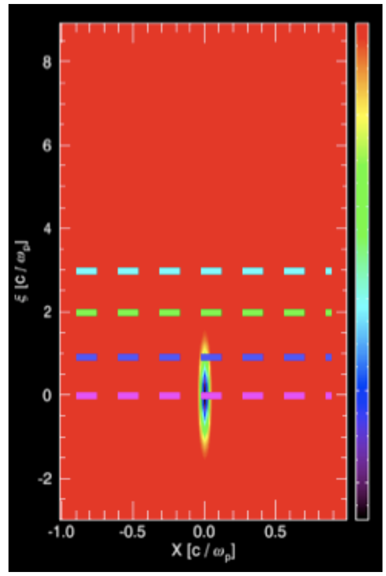


Figure 1 shows the transverse focusing force at the different marked places in the beam. The width of the maroon box covers the transverse extent of the Gaussian beam. It is apparent from this plot that after the beam has passed, the motion of the ions causes the focusing force to become significantly stronger: even though it still appears linear, its slope has more than doubled.

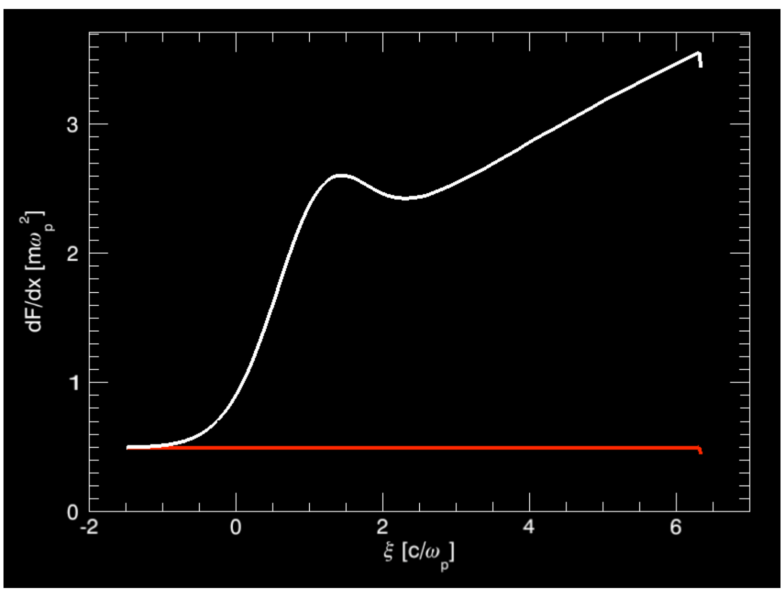


Figure 2. Focusing force strength as a function of ξ. (“spring constant” of the plasma aft of the beam). Plotted in RED is the stationary ion approximation. Plotted in WHITE is the simulation with mobile ions.

In figure 2 the slope of the focusing force spring constant as a function of ξ , i.e. at different locations within the beam, is plotted (left is the head and right is the tail). This plot shows that although ion collapse can occur after the the driver passes it still is an issue for the trailing beam. Tightly focused trailing beams can also cause further collapse and modify the focussing forces within itself. Ion collapse can therefore be an issue for laser wakefield acceleration as well.

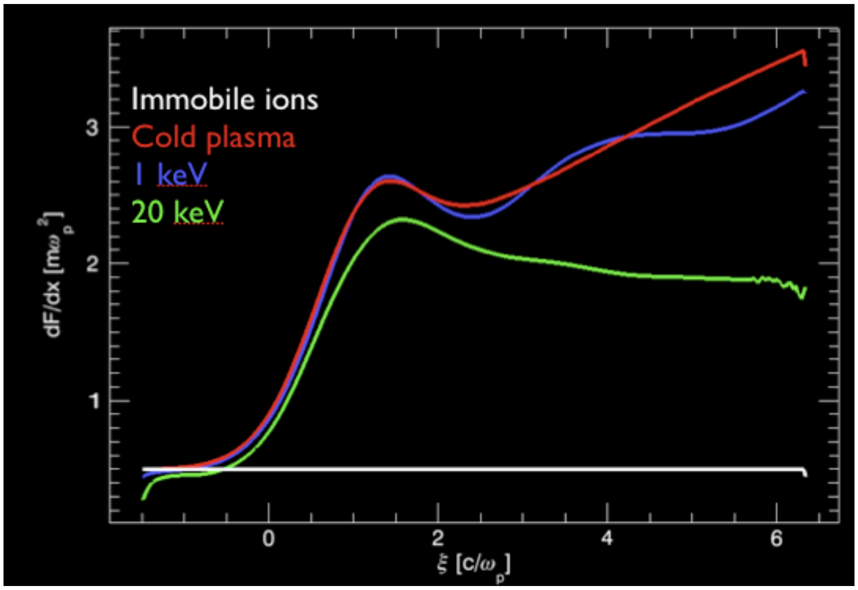


Figure 3. The effect of plasma temperature on plasma spring constant, as a function of ξ

I wanted to investigate the effect of ion temperature on the ion motion because in an accelerator context, because at the rep rate of a future accelerator it may not be possible to extract the energy deposited in the source between each drive beam. As a result the plasma may settle at some temperature. This temperature is not likely to be greater than 1 keV.

The idea was that the gas pressure of the hot plasma would prevent some of the collapse, and also that the random thermal motion would wash out the effect more quickly than for a cold plasma. Only the latter proved true. As shown in figure 3, even for the a temperature of 20 keV (implausibly high for accelerators) the effect on the effective spring constant for the focusing force is not dramatic.

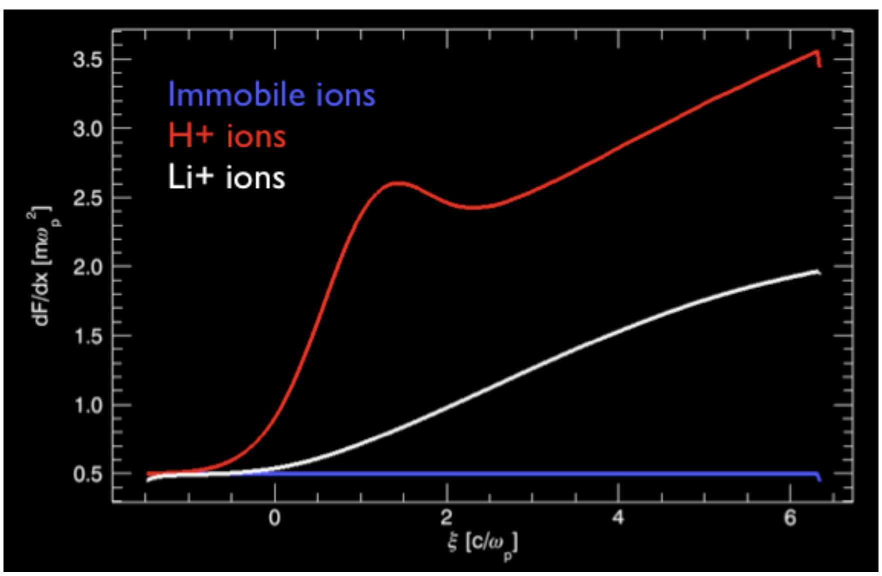


Figure 4. Spring constant of the plasma for ion species of different mass ($m_{Li+}=7m_{H+}$), as a function of ξ

Finally, I looked at the effect of different masses of ions, specifically at what happens when lithium is used instead of hydrogen. It is to be expected that inertia will reduce ion motion, but it is important to find out by how much.The effect is significantly reduced. One can anticipate that with heavier ions, motion can be contained better.

V. Resolution problems and scaling of the results

The results presented above are not done for sufficient resolution to be quantitatively accurate. The highest resolution allowed by QuickPIC on the number of processors I used was not sufficient enough to see the actual structure of the ion buildup.

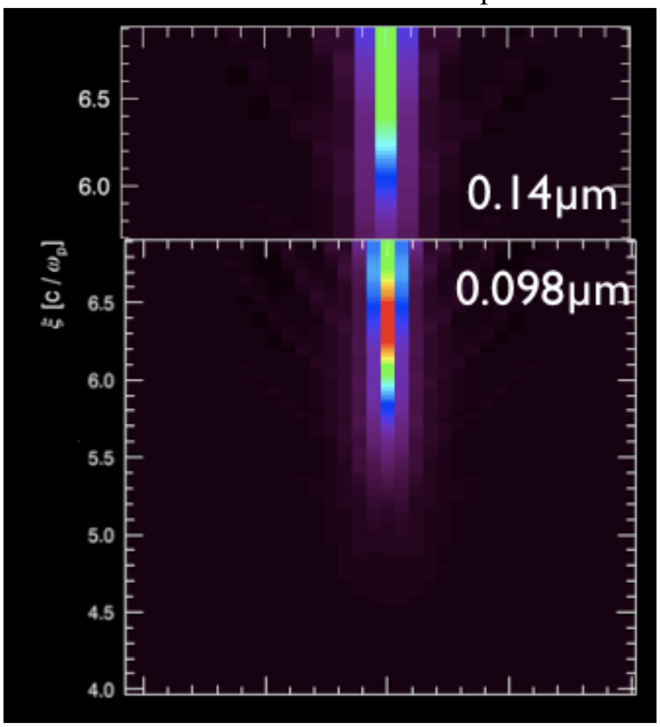


Figure 5. Plasma density at different resolutions; same scale

The above plot shows that when resolution is improved, we see that the fine details of ion buildup change significantly. This implies also that the slope of the focusing force will change with increased resolution. On the other hand, since Gauss's law (used for the electric field) depends not on the charge density itself but on its volume integral, pertinent quantities should be roughly the same except very close to the axis.

Regardless of the above, and even if the results are quantitatively inaccurate, the scaling of the focusing forces (for example, all of the plots with respect to ξ) nevertheless illustrate the effects of mass and thermal motion.

VI. Conclusions

I gained some insight into how ions move due to a high density driving beam and how this affects the focusing forces on the beam and any trailing beams.

Thermal pressure for temperatures as high as 10keV did not seem to have an insignificant effect on how ion collapse modifies the focusing forces. On the other hand, higher mass cam significantly reduce the severity of the ion collapse.

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VIII. References

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