Thermo-Physical Modeling of W.I.S.E. Observed Asteroids; Determining the Diameter and Investigating the Yarkovsky Effect

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August 29, 2011

Abstract

The near Earth asteroid population is continuously depleted through collisions and by ejection into the inner solar system while simultaneously being replenished by main belt asteroids that strayed into orbital resonances with Jupiter and Mars. The standard collisionary model for the injection of these asteroids into the resonances contradicts observations, so it is believed that the Yarkovsky and YORP (Yarkovsky-O'Keefe-Radzievskii-Paddack) effects are responsible. In this paper a method for determining the pertinent properties of asteroids in order to simulate the change in orbital angular momentum due to these effects is laid out along with results using WISE (Wide-field Infrared Survey Explorer) observations. Using thermophysical models to predict the flux from an asteroid at the WISE wavelengths estimations of the asteroid's diameter, north pole orientation, and thermal inertia parameter were obtained. This allowed for the calculation of the imbalance in the emission of thermal photons and the subsequent net momentum transfer. Preliminary examinations using typical main belt asteroids 243516 and 112446 yielded tight constraints on their diameter, probable regions for north pole orientation, and approximate values for the thermal inertia parameter. Using these results the corresponding timescales for Yarkovsky and YORP effects to move the asteroids 1% of their orbital radii were determined.

1 Introduction

The largest population of Asteroid's in the inner solar system is the main belt, located between the orbits of Mars and Jupiter (i.e., between 2.0 AU-3.3 AU). There is a smaller population of solar system bodies known as near-Earth objects (NEOs), which are asteroids and comets that have a perihelia, $q, \leq 1.3$ AU. Numerical simulations indicated that NEOs have mean dynamic lifetime of several millions of years and that only 1% end up as Earth impactors, the most probable end state is impacting the Sun or ejection from the inner Solar System via a close encounter with Jupiter (Gladman et al. 1997). An interesting aspect of this population is that cratering rates on the Earth and the Moon have been approximately constant over the last 3.2 billion years (McEwen et al. 1997). This implies that the mechanism responsible for the upkeep of the NEO population has been constant over this time. The main belt is the most probable source for consistently resupplying the NEO population.

In 1983 Wisdom showed that secular and mean-motion resonances can account for the transport of main belt asteroids to the near Earth population. Once entering a resonance the asteroid's orbital eccentricity can be increased enough to lead to gravitational interactions with planets which can extract the asteroid from the resonance and deposit it into a new, Earth-crossing orbit in only one million years.

Conventional wisdom has gravitational interactions as the sole mechanism for injecting asteroids into the resonances, however certain contradictions with observations have lead some to consider other models. The collisionary gravitational model incorrectly predicts near Earth asteroids' size, velocity, and spin-rate frequencies, as well as their cosmic ray exposure (CRE) times, which is a measure of time between collisions. It is believed that when the diurnal Yarkovsky effect along with its seasonal counter-part the YORP effect are taken into account no such troubles arise (Bottke et al. 2006).

1.1 Yarkovsky & YORP effects

The Yarkovsky effect is a small force on bodies orbiting the Sun which is produced when they absorb solar energy, heat up, and then reradiate the energy as thermal emission after a delay. Because the hottest part of a body will radiate away more energy than any other part it will also experience the largest transfer of momentum as it emits photons. This imbalance in momentum transfer will create a slight force on any body that has an uneven temperature distribution in the direction opposite to the hottest point.

The Yarkovsky effect can be broken down into two components, diurnal and season. The diurnal effect arrises from the asteroid's daily rotation and was originally conceived of by the Russian civil engineer Ivan Yarkovsky around the year 1900. The seasonal component or YORP effect is dependent on the mean motion of the body around the Sun and its existence was deduced several times throughout the next century. The YORP effect changes the spin of an asteroid, while the Yarkovsky effect changes the orbit.

The seasonal, YORP effect is caused by the fact that the seasonal heating of northern and southern hemispheres of solar system bodies peaks at a point somewhat after the solstices. Precise modeling of the YORP effect is difficult because it is dependent on specific details of the bodies shape.

The diurnal Yarkovsky effect is caused by the fact that for all rotating bodies in the solar system the sub-solar point is not the hottest point on the surface on a daily basis. Instead due to thermal inertia, a lag in energy absorption and emission, the hottest point is on the afternoon-side. The force that arises is small but capable of producing a secular change in semi-major axis, and to a lesser extent eccentricity. The direction of the force is determined by the north pole orientation. If the object is a prograde rotator (i.e., its north pole has positive declination) then it will experience an increase in orbital angular momentum where as retrograde rotators will lose orbital angular momentum. The magnitude of the force depends on the distance the body is from the Sun, the angle of the north pole with respect to the orbital plane, the body's size, shape, it's thermal properties, and how fast it is rotating.

1.2 WISE

NASA's WISE telescope provides the ideal data to study the Yarkovsky effect. WISE observed the entire sky in four channels that were centered at 3.4, 4.6, 12, and 22 μm. WISE had a 47 arcminute field of view and with 6" resolution (12" for band 4) it observed much of the main belt and near Earth population of asteroids multiple times and with high signal-to-noise ratios (Wright et al. 2010). A key aspect of the WISE observations is that WISE was able to observe many asteroids at two different points in their orbits around six months apart. Multiple observations enables a comparison of an asteroid's thermophysical properties at different points along its orbit, which allows for constraints on important factors for the Yarkovsky force.

In this paper a method for determining these necessary characteristics for asteroids that were observed by WISE is presented. The results of this method applied to two main belt asteroids, 243516 and 112446, and a preliminary study of the Yarkovsky force is introduced.

2 Methods

Initial analysis of the asteroids focused on their light curves. The light curve provides information on the asteroid's rotational frequency, Ω, and its shape.

The rotation frequency plays a key role in the diurnal Yarkovsky effect because it directly effects the distribution of heat on the surface of the asteroid. As indicated by the fact that as the rotational frequency increases to infinity the Yarkovsky force diminishes to zero because the body no longer experiences a daily variation of temperature and the distribution becomes azimuthally symmetric. And conversely, for a non-rotating body the sub-solar point is at the highest temperature and as a result the body experiences only a positive radial force. To determine Ω the light curve of the asteroid was fit to a second order sine function:

$$A_1 \sin \Omega t + B_1 \cos \Omega t + A_2 \sin 2\Omega t + B_2 \cos 2\Omega t + C$$

A highly precise value of Ω is not necessary because it will be varied in conjunction with other parameters later. The second order sine function accounts for variations in the shape of the asteroid enough to constrain the period to a suitable region.

The light curve also gives indications as to the asphericity of the body. A perfectly spherical object will have a flat rotation curve because the same amount of area is exposed to the sun and the observer as it rotates, however a deviation from spherical will give the light curve a sinusoidal appearance. Thus, the amplitude of the light curve oscillations is related to the amount the body differs from a sphere. For asteroids whose light curves exhibited small variations from the average an ellipsoidal shape was assumed with an axis ratio determined by the magnitude of the amplitudes.

2.1 Thermophysical Modeling

Once estimates were obtained for the rotation frequency and the shape of the asteroid modeling of the thermophyical properties of the asteroid was possible. The rotating cratered asteroid model as written by Dr. Edward Wright in 2007 assumes only vertical heat conduction across the surface of an entirely cratered spherical, elliptical, or egg-shaped asteroid. The model determines the temperature as a function of time at each by solving:

$$C\rho \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \tag{1}$$

Where C is the specific heat per unit mass, ρ is the density, and κ is the thermal inertia. This equation simplifies to:

$$\Theta^2 \frac{\partial y}{\partial \theta} = \frac{\partial^2 y}{\partial \omega^2} \tag{2}$$

Where $y = \frac{T}{T_\circ}$ is the temperature relative to the equilibrium temperature $T_\circ = (\frac{(1-A)L_\odot}{4\pi\epsilon\sigma R^2})^{1/4}$, $\theta = \Omega t$, and $\omega = \epsilon\sigma T_\circ^3 z/\kappa$. The coefficient:

$$\Theta = \frac{\sqrt{\kappa\rho C\Omega}}{\epsilon\sigma T_\circ^3} = \frac{\sqrt{\kappa\rho C\Omega R^3}}{(\epsilon\sigma)^{1/4}[(1-A)L_\odot/4\pi]^{3/4}} \tag{3}$$

is the thermal inertia parameter, which is a dimensionless measure of the importance of the thermal inertia on the temperature of the body. ϵ in this equation is the asteroid's emissivity. The thermal inertia parameter at 1 AU, Θ_1, contains the necessary thermal properties of the asteroid to determine the heat distribution. By varying Θ_1 different thermal properties can be tried and then scaled to match the distance to the sun, $\Theta = \Theta_1 R^{3/2}$ (Wright 2007).

By specifying the shape, the sub-solar latitude, the sub-earth latitude, the Earth's local hour angle (the time of day on the asteroid that an Earthbound observer is seeing), and Θ_1 the rotating cratered asteroid model computed the fluxes as seen by an observer at the Earth's location in WISE bands 2,3, and 4 (W2, W3, W4), which are most suited to asteroid observations. By default the calculations are done for a 1 km asteroid at a 1 AU from the Earth. Therefore to model a particular asteroid the diameter needs to be scaled, the distance from the Sun and the Earth need to specified, as well as Θ_1 and the north pole orientation, which determines the three necessary angles.

2.2 Determining the Diameter for a given North Pole and Thermal Inertia Parameter

Given a north pole orientation (i.e., right ascension and declination) the sub-solar and sub-earth latitudes are specified by:

$$SLAT = \arcsin(\vec{NP} \cdot \vec{Sun}) \tag{4}$$

$$ELAT = \arcsin(\vec{NP} \cdot \vec{Earth}) \tag{5}$$

Where the three vectors describe the north pole, from the asteroid to Sun, and from the asteroid to the Earth. The Sun and Earth vectors are obtained from the date and position of the asteroid when it was observed. The Earth's local hour angle is the angle between the Earth's meridian and the solar meridian (noon on the asteroid), positive is the afternoon side and negative is the morning side. It is calculated by letting $\vec{u}$ and $\vec{v}$ be the components of the Sun and Earth vectors in the plane perpendicular to the north pole, $\vec{u} = \vec{Sun} - (\vec{NP} \cdot \vec{Sun})\vec{NP}$, $\vec{v} = \vec{Earth} - (\vec{NP} \cdot \vec{Earth})\vec{NP}$, and then determining the angle between the two vectors and the appropriate sign.

With the necessary angles for a given north pole orientation the predicted fluxes were calculated for a 1 km asteroid by specifying Θ_1. With the fluxes on hand the diameter was scaled to match the observed magnitudes using a weighted mean. When only considering one band the equation, $m_1 - m_2 = -2.5 \log F_1/F_2$, leads to: $D_{obs} = 10^{(\bar{m}_{pred} - \bar{m}_{obs})/5}$ where $\bar{m}_{pred}$ is the predicted magnitude averaged over 1 revolution in a given band for $D_{pred} = 1$ km and D_{obs} is the diameter that best matches the observed average magnitude, $\bar{m}_{obs}$. When taking into account two observations separated by six months in three bands, W2, W3, and W4, and weighting them with one over their respective uncertainties squared, the diameter for the given selection of north pole right ascension, and declination $(\alpha_{np}, \delta_{np})$, and Θ_1 becomes:

$$D(\alpha_{np}, \delta_{np}, \Theta_1) = 10^{\frac{\Sigma_{b,m}[w_{b,m} \triangle \bar{m}_{b,m}]}{5(\Sigma_{b,m}[w_{b,m}])}} \tag{6}$$

Where the sum is over bands and months. The weights for each band in each month are $w_{b,m} = 1/\sigma_{b,m}^2$ and $\triangle \bar{m}_{b,m} = \bar{m}_{pred,b,m} - \bar{m}_{obs,b,m}$. With the new diameter the rotating cratered asteroid model can be run again to determine the average magnitudes in W2, W3, and W4, as well as the root-mean-square amplitudes of their respective light curves.

These predicted values at the best fit diameter allow for a comparison to the observed data. By calculating the χ^2 for the average magnitude in each band and in each of its observed months, as well as the log of the difference of the ratios of the amplitudes in each month in each band a quantitative measure of the fit of the model is obtained for the choices of α_{np}, δ_{np}, and Θ_1.

$$\chi^2_{\alpha_{np},\delta_{np},\Theta_1} = \Sigma_{bands,months}\left(\frac{\bar{m}_{obs,b,m} - \bar{m}_{pred,b,m}}{\sigma_{b,m}}\right)^2 + \Sigma_b\left(\frac{\log\left(\frac{amp_{pred,m_1,b}}{amp_{pred,m_2,b}} / \frac{amp_{obs,m_1,b}}{amp_{obs,m_2,b}}\right)}{\sigma_b}\right)^2 \tag{7}$$

Where *amp* is the root mean square amplitude of either the observed or predicted values in a given band for a given month.

2.3 Parameter Space

By systematic sampling of the possible choices for the necessary parameters, α_{np}, δ_{np}, and Θ_1, it is possible to determine the most likely choices for these parameters and to determine the most likely diameter of the asteroid. The north pole right ascension, α_{np}, runs from 0 to 360 degrees, and the north pole declination, δ_{np}, runs from -90 to 90 degrees. Possible values for Θ_1 were determined by using the rotational frequency from the second order sinusoidal fit, and realistic values for the other parameters. The resulting range for Θ_1 was from 0.4 on the low end up to around 50 for the upper bound, although values above or below are not impossible.

In order to bin the possible choices for α_{np} and δ_{np} with equal area bins Hierarchical Equal Area isoLatitude Pixelization of a sphere (HEALpix) was used. Θ_1 values were distributed on a logarithmic prior. For each point in the parameter space a χ^2 value was calculated along with the corresponding diameter. This allowed for determination of the most likely parameter combinations and the weighted-mean diameter using $e^{-\chi^2/2}$ for the weight.

$$D_{weight} = \frac{\Sigma_i(D_i * e^{-\chi_i^2/2})}{\Sigma_i(e^{-\chi_i^2/2})} \tag{8}$$

Where D_i and χ_i^2 correspond to the values obtained at the i^{th} point in the parameter space.

2.4 Yarkovsky Force Model

In order to model the Yarkovsky force the temperature and placement of the hottest point on the asteroid must be known. In order to determine the temperature distribution the effective temperature of the asteroid as seen from observers placed at HEALpix pixels on a 1 AU sphere around the asteroid. With the effective temperature at each HEALpix pixel the force in that direction can be calculated because the force from a radiating blackbody is directly related to the luminosity by $F = L/c$. The luminosity of a blackbody is given by $L = \sigma T^4_{effective}$, thus the force is proportional to T_e^4. By summing the force vectors from each HEALpix pixel the net force experienced by the asteroid can be determined, however we are only interested in the change in angular momentum which means we want the torque from that force, $\Gamma = \vec{D}_{sun} \times \vec{F} = \epsilon D_{sun} F$. Where ϵ is the component of the force that is in the orbital plane and perpendicular to the Sun vector. A positive ϵ term means there will be an increase in angular momentum and a negative ϵ means there will be a loss in angular momentum.

$$\epsilon = \frac{\sum_i \hat{n}_i \cdot \hat{v}\, T^4_{e,i}}{\sum_i T^4_{e,i}} \tag{9}$$

Where T_e is the average effective temperature over one rotation when looking along direction $\hat{n}$ for the i^{th} HEALpix pixel, and $\hat{v}$ is the unit vector in the orbit plane $\perp$ to the Sun line.

The angular momentum of the asteroid is given (assuming a circular orbit) by

$$L_{orb} = \frac{4\pi}{3} r^3 \rho \sqrt{GM_\odot D_{sun}} \tag{10}$$

Asteroid	Diameter	Standard Deviation
243516	3.37247324 km	0.171120271 km
112446	4.53790474 km	0.263292372 km

Table 1: Weighted-mean Diameter results as determined by equation 8 along with corresponding standard deviation

Then, to first order, the time scale for the Yarkovsky effect to change D_{sun} by 100% is given by

$$\begin{aligned} t_Y = \frac{1}{2}\frac{L_{orb}}{T} &= \frac{(4\pi/3) r^3 \rho \sqrt{GM_\odot D_{sun}}}{2\epsilon L_\odot r^2/(4 D_{sun} c)} \\ &= \frac{8\pi}{3}\frac{r\rho c}{\epsilon L_\odot}\sqrt{GM_\odot} D_{sun}^{3/2} \end{aligned} \tag{11}$$

which evaluates to:

$$t_Y = 17 \times 10^9 \text{yrs} \times \frac{r}{1.6 \text{ km}} \frac{\rho}{2 \text{ gm/cc}} \frac{0.1}{\epsilon} \left(\frac{D}{2.5 \text{ AU}}\right)^{3/2} \tag{12}$$

Therefore the approximate time necessary to move an asteroid 1 % of its orbit and possibly into an orbital resonance is 0.01 t_Y.

3 Results

3.1 Diameter, North Pole, and Thermal Inertia Parameter

In preliminary testing of the described method two standard main belt asteroids observed by WISE on two occasions were used. Once specifying the positions of the asteroids and corresponding dates of the observations the three parameters, α_{np}, δ_{np}, and Θ_1, were varied 5376 times. This was done by using a HEALpix resolution of three, which implies the sphere was broken down into $12 * 4^3 = 768$ equal area bins, and trying seven values of Θ_1 which were distributed on a logarithmic prior with values ranging from $10^{-1/3} \approx 0.46$ to $10^{5/3} \approx 46.42$.

With the 5376 diameter and χ^2 values the weighted-mean diameters for both asteroids were calculated. The results are listed in table 1. The most likely north pole orientations were calculated for each of the seven thermal inertia parameters that

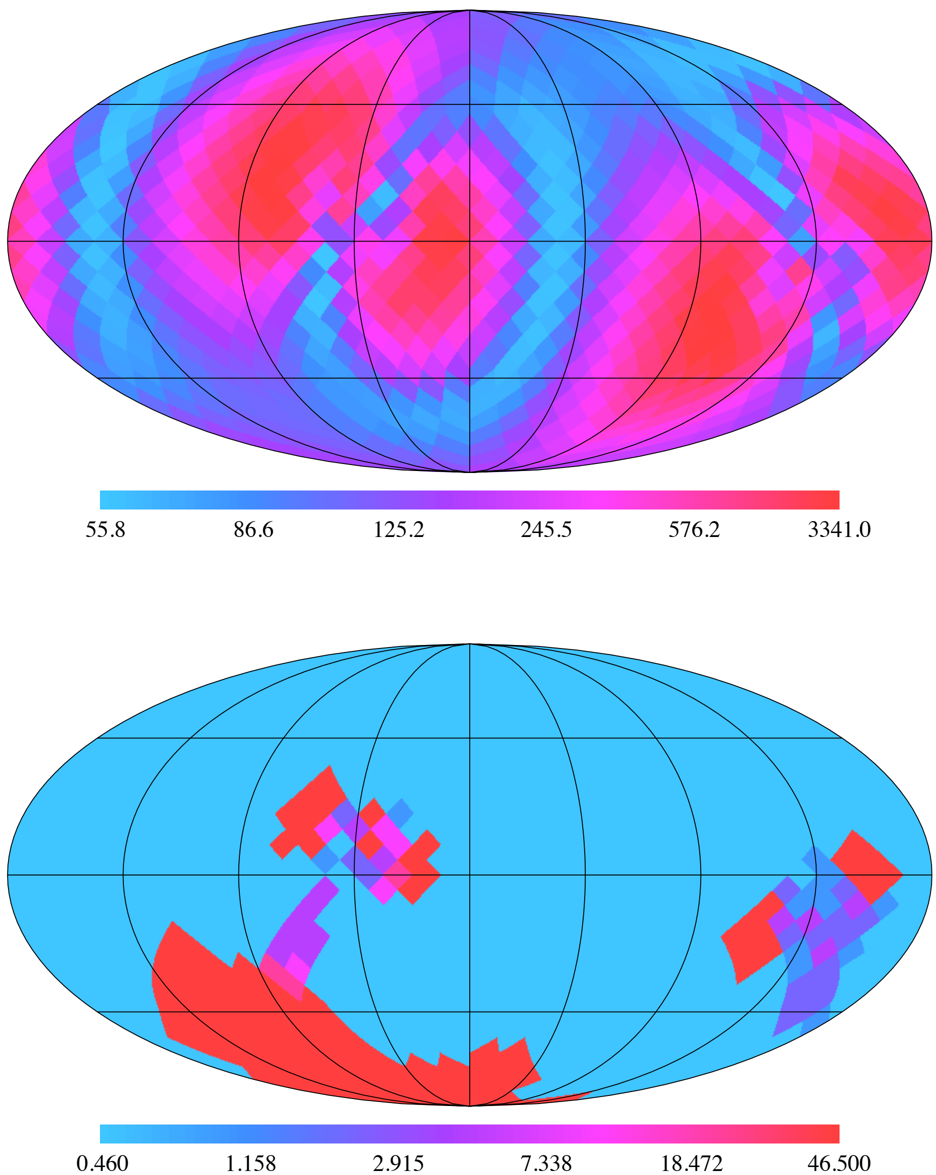

Figure 1: (top) Average χ^2 for each north pole orientation of asteroid 243516. (bottom) Best fit thermal inertia parameter for a each north pole orientation of asteroid 243516.

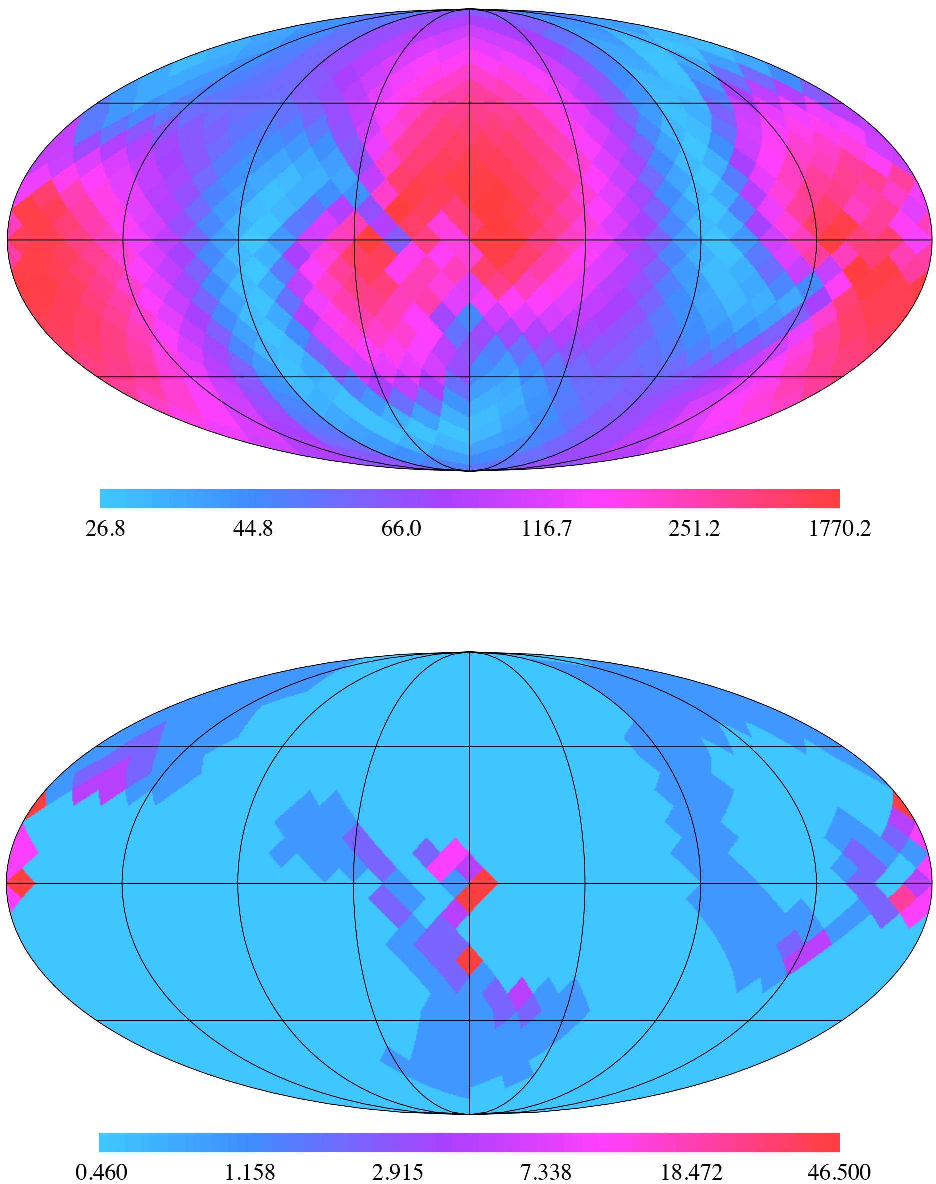

Figure 2: (top) Average χ^2 for each north pole orientation of asteroid 112446. (bottom) Best fit thermal inertia parameter for a each north pole orientation of asteroid 112446.

were tried. The top maps in figures 1 and 2 show the χ^2 for each north pole orientation tried averaged over the thermal inertia parameters for 243516 and 112446 respectively. The bottom maps in figures 1 and 2 show the best thermal inertia parameter for a given north pole arrangement.

3.2 Yarkovsky Force timescale

The results of the Yarkovsky force analysis are twofold. First, the ϵ term was calculated, which was done using the lowest HEALpix resolution, 12 points. The effective temperatures as seen at these twelve points were used to calculate ϵ. Second, a calculation of the time necessary to move the asteroid 1% of its orbit. This was done for each of the points in the parameter space. Figure 3 shows a map of the calculated ϵ for the best average choices of Θ_1 for each asteroid. Tables 2 and 3 show the timescales and ϵ for five of the best-fit combinations of parameters.

The calculated time scales in order to move the asteroid 1 percent of its orbit range from almost infinite (negligible Yarkovsky effect) to a time of approximately 150 million years. These time scales are on the order of the CRE times that have been observed for near-Earth asteroids.

4 Conclusions and Future Work

The preliminary results presented indicate the potential success for the method laid out in section 2. Using WISE observations it is possible to determine an asteroid's thermophysical properties. Diameter estimates determined using the presented method agree with values obtained using Harris' 1998 Near Earth Asteroid Thermal Model. Although the north pole right ascension and declination were not determined as definitively as the diameter, by increasing the number of points in the parameter space a tighter constraint will be achieved. Likewise by increasing the number of thermal inertia parameters tried all of the thermophysical properties necessary to model the Yarkovsky effect will be obtained.

In conjunction with a finer grid in the parameter space a more detailed model of the Yarkovsky force will be possible. The first-order estimates presented here indicate that the magnitude of the force is enough to consistently move multi kilometer sized asteroids into orbital resonances in an amount of time that agrees with observations. In future studies of the Yarkovsky effect a higher resolution will be used in calculating ϵ which will allow for a more precise estimate for the torque applied. Additionally, by modeling the Yarkovsky effect around the whole orbit instead of averaging it across one orbit a more accurate model will be obtained.

By increasing the number of points tried in the parameter space, increasing the detail of the Yarkovsky model, and by applying the method to many more asteroids an accurate picture of the impact of the Yarkovsky effect on the orbital evolution of asteroids, on an asteroid by asteroid basis, will be obtained. With the result of this knowledge being an understanding of the upkeep of the near Earth asteroid population.

5 Acknowledgments

I would sincerely like to thank Dr. Ned Wright for mentoring me on this project and for all of his work to make WISE a reality. In addition I would like to thank Sean Lake for his help with programming difficulties and the UCLA Department of Physics and Astronomy for providing its facilities.

This publication makes use of data products from the Wide-field Infrared Survey Explorer, which is a joint project of the University of California, Los Angeles, and the Jet Propulsion Laboratory/California Institute of Technology, funded by the National Aeronautics and Space Administration.

This project was supported by grant NSF-PHY 0850501 from the National Science Foundation as part of the University of California Los Angeles's Physics and Astronomy Research Experience for Undergraduates.

References

Bottke, W. F., et al. 2006. The Yarkovsky and YORP Effects: Implications for Asteroid Dynamics. Annual Review of Earth and Planetary Sciences Vol. 34: 157-191

Gladman, B.J., et al. 1997. Dynamical lifetimes of objects injected into asteroid belt resonances. Science 277:197201

Harris, A. W. 1998. A Thermal Model for Near-Earth Asteroids. Icarus 131, 291-301.

McEwen AS, et al.. 1997. The Phanerozoic impact cratering rate: Evidence from the farside of the Moon. J. Geophys. Res. 102:923142

Wisdom J. 1983. Chaotic behavior and the origin of the 3/1 Kirkwood gap. Icarus 56:5174

Wright, E. L., et al. 2010. Mission Description and Initial On-orbit Performance. astro-ph/1008.0031v2

Wright, E. L. 2007. Comparing the NEATM with a Rotating, Cratered Thermophysical Asteroid Model. astro-ph/0703085v2

α_{np}	δ_{np}	Θ_1	Diameter	ϵ	Yarkovsky Time scale (m yrs)
33.75	14.4775105	10.	3.30712509	0.053594172	327.81738
-129.374985	10^{-6}	0.464158893	3.28109789	0.0137541508	1267.31431
33.75	14.4775105	4.64158869	3.30222034	0.0535508394	327.596098
-129.374985	9.59406757	0.464158893	3.37609315	-0.000727265608	24661.5467
-123.749977	14.4775105	0.464158893	3.46911716	-0.00630929135	2921.03887

Table 2: Best parameter combinations with corresponding ϵ and Yarkovsky Time scale to move the orbit by 1% for asteroid 243516

α_{np}	δ_{np}	Θ_1	Diameter	ϵ	Yarkovsky Time scale (m yrs)
78.75	-14.4775105	0.464158893.	4.45379019	-0.0497499034	475.594103
78.75	-24.6243153	0.464158893	4.49633217	-0.0513052829	465.58097
78.75	-4.78019238	0.464158893	4.42197275	-0.047028821	499.517739
73.125	10^{-6}	0.464158893	4.28991461	-0.0440220609	517.698824
78.75	-35.6853333	0.464158893	4.5543642	-0.051447548	470.285952

Table 3: Best parameter combinations with corresponding ϵ and Yarkovsky Time scale to move the orbit by 1% for asteroid 112446

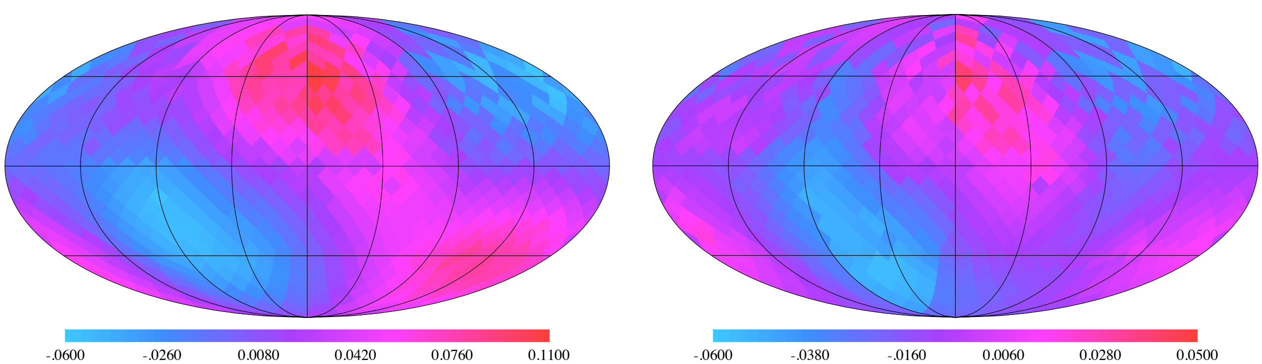

Figure 3: Map of ϵ values for each north pole arrangement and a thermal inertia parameter $\Theta_1 \approx 0.46$. The map on the left is for asteroid 243516 and the map on the right is for asteroid 112446.