

Sunyaev-Zeldovich Contributions from Early Supernova Winds

Sarah Benjamin*

Mentor: Steven Furlanetto

Department of Physics and Astronomy, UCLA

September 5, 2011

Abstract

The purpose of this research was to determine the contributions to the Sunyaev-Zeldovich effect from supernova winds at high redshift. In order to accomplish this, the shells were modeled individually to study the dependence of their behavior on the basic input parameters of the system, such as initial redshift and initial dark matter halo mass, especially at asymptotic bounds. Finally, the Compton y -parameter of the supernova winds can be found using the modeled results for the amount of energy lost via Compton cooling relative to the total amount of energy input by all the supernovae. The final calculation for the y -parameter is still in progress, but the model behaves reasonably for modifications in parameters.

1 Introduction

The Sunyaev-Zeldovich (SZ) effect is the presence of thermal angular fluctuations in the Cosmic Microwave Background Radiation (CMBR) that arise from inverse Compton scattering of CMB photons in hot gas. The current bound on the known anisotropy of the CMBR from the Cosmic Background

*Undergraduate at Carnegie Mellon University

Explorer (COBE) Far-Infrared Absolute Spectrophotometer (FIRAS) puts the maximum SZ effect Compton parameter at $y < 1.5 \cdot 10^{-5}$ (Fixsen et al. 1996).

Early star formation occurred rapidly and resulted in massive stars. At the end of their lifetimes, these star supernova, each one producing about 10^{51} ergs, which means that for a galaxy with a given fraction of star formation f_{star} and a galaxy mass that is a fraction f_{gal} of all the baryonic matter in the halo, the total amount of energy generated is,

$$E_{SN} = \frac{10^{51} \text{ ergs}}{M_{sn}} f_{esc} f_{star} (f_{gal} \frac{\Omega_b}{\Omega_o} M_{halo}) \quad (1)$$

In this case, M_{SN} represents the mass of star formation required per supernova and f_{esc} determines how much energy escapes the galaxy. For the purposes of this model, these two values are combine into one parameter $\nu_{SN} = M_{SN}/f_{esc}$ which governs all parameters of the generated supernova energy.

These supernovae individually do not produce enough energy to be significant to the surrounding universe. However, the beginning stages of star formation within early galaxies create a sufficient amount of supernovae over a short period of time that they can be approximated as a constant luminosity driving local material outwards. The result is a thin spherical shell of material that is propelled by the supernova energy out of the central galaxy and significantly past the virial radius of the halo. As this shell expands, the interior cools through radiation, which at high redshift is dominated by Compton scattering with the CMBR. This expansion can significantly alter the local area as the shell ionizes the surrounding matter and distributes metals throughout.

It has been proposed that these galactic winds produced by massive supernovae may contribute a significant portion of the SZ effect through Compton cooling of the interior (Furlanetto & Loeb, 2003). The purpose of this research was to determine whether this value is in fact significant around the time of re-ionization by direct modeling of the behavior of these winds. A previous calculation of this value was made analytically using parameters determined from the Wilkinson Microwave Anisotropy Probe (WMAP) which matched the COBE bound with $y \sim few \cdot 10^{-6}$ (Oh, Cooray, & Kamionkowski, 2003).

All models are generated using the standard Λ CDM cosmological parameters of $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_b h^2 = 0.019$, $\sigma_8 = 0.9$, $n = 1$, $h = 0.7$.

Also, the redshift range defining the "early" universe are constrained to be between $z = 15$ and 7 .

2 Supernova Wind Model

Because behavior of the winds and each individual supernova within the galaxy is very difficult to model, I assume that the net input is a luminosity that is a function of the age and total mass of the galaxy, and that the ejected material is already blown into a thin spherical shell as it moves out of the galaxy. While star formation is still occurring, more mass and energy is added to the shell from the central galaxy. As the shell continues moving outwards it also accretes additional baryonic matter through perfect inelastic collision with the surrounding matter. The kinetic energy can either be immediately radiated away, referred to as shock cooling, or some fraction f_d is radiated back to the interior of the shell where it contributes additional energy to the expansion. The majority of the mass swept up by the winds remains in the outer shell, but a small fraction f_m leaks into the interior. The shell expands until it's velocity drops to that of the Hubble flow, at which point it expands with the rest of the universe.

2.1 System Initialization

The initialization of the shell is dependent on the approximate parameters that govern the distribution of matter within the halo and the behavior of the supernovae themselves. Within the halo, dark matter is assumed to be in a singular isothermal sphere, a $1/r^2$ density profile. Additionally, there is the f_{sw} parameter, which defines what fraction of the total galaxy mass will eventually be added to the shell during the process of star formation.

In the simplest case, luminosity is constant for the period where $t < t_{sf}$ and then shuts off instantly, without decay. This constant luminosity is then,

$$L_{sn} = \frac{E_{sn}}{t_{sf}} = 3.15 \cdot 10^{43} \left(\frac{f_{star} f_{gal} \frac{\Omega_b}{\Omega_o} M_{halo}}{t_{sf} \nu_{sn}} \right) \text{ ergs } s^{-1} \quad (2)$$

There are two initialization methods used based on the concentration of baryons within the central galaxy. In both cases, since, as mentioned earlier, it is difficult to model wind behavior within the galaxy, the shell is initialized

with a starting radius equal to that of the galaxy. This is the scale length

$$R_d = \frac{\lambda}{\sqrt{(2)}} R_{vir}, \quad (3)$$

where R_{vir} is the virial radius of the halo, and $\lambda = 0.05$ (Furlanetto & Loeb, 2003).

If it is assumed that the baryons are not highly concentrated in the center galaxy, then the initial galaxy and shell masses are defined by set parameters f_{gal} and an additional f_{shell} . The time since the beginning of star formation is determined from the density and luminosity according to the self-similar wind-fed solution of Ostriker & McKee (1988), and the initial velocity and pressure result in turn from this time in combination with the radius and mass. In this case, the remaining baryons not within the galaxy also match the singular isothermal sphere density profile.

However, for more accurate comparison between these results and previous work, the results shown later on were all generated assuming that all baryons within the halo collapsed into the central galaxy ($f_{gal} = 1.0$). In this case, an initial velocity is derived from the starting parameters (Springel & Hernquist 2002)

$$v_{init} = 7.23 \cdot 10^{-6} \left(\frac{f_{star}}{\nu_{sn} f_{sw}} \right)^{\frac{1}{2}} \text{ kpc yr}^{-1} \quad (4)$$

With the standard set of parameters used for all the shells modeled for this paper (Table 1), this is $v_{init} = 222.961 \text{ km/s}$. Time elapsed since the beginning of star formation is calculated assuming constant velocity up until this point, and it is assumed that all energy already output by the supernovae is divided evenly between thermal and kinetic energy (which in turn determines the shell mass).

Table 1: Standard Parameters for the Supernova Wind Model

f_m	0.1
f_{star}	0.1
ν_{sn}	$126 M_{\odot} / 0.25 = 504 M_{\odot}$
f_{sw}	$2 * f_{star} = 0.2$
t_{sf}	10^7 years

Outside of the halo, it is assumed that the baryon and dark matter distribution very quickly returns to uniform at the critical density. Therefore, the final function for u , the velocity of matter added to the shell is,

$$u = \begin{cases} 0 & r \leq r_{vir} \\ Hr & r > r_{vir} \end{cases}$$

2.2 Shell Expansion

Once initialized, the expansion of the shell can be described using the following three equations:

$$\dot{m} = 4\pi r^2 \rho(\dot{r} - u) + f_{sw} \frac{M_{gal}}{t_{sf}} \Theta(t_{sf} - t) \quad (5)$$

$$\ddot{r} = -G \frac{M_{enc}}{r^2} + \frac{4\pi r^2}{m} (p - p_{ext}) + \Omega_\Lambda(z) H(z)^2 r - \frac{\dot{m}_{r>r_v}}{m} (v - u) \quad (6)$$

$$\dot{p} = \frac{L}{2\pi r^3} - 5 \frac{\dot{r}}{r} p \quad (7)$$

The first equation (5) describes the mass that is added to the system from both sources - the central galaxy and the surrounding universe. The first term represents the portion accreted through inelastic collision, and results from the fact that the critical density of baryonic matter is dependent on redshift. The second is the constant addition of matter from the central galaxy, and shuts off when $t > t_{sf}$.

The second equation (6) describes the acceleration, and is the result of the four forces acting on the shell during its expansion. The first term results from gravity, the second is the outwards force due to pressure, $F = pA$, where outside the shell, the shell experiences a resistant external pressure from the surrounding gas. This pressure is assumed to be that of an ideal gas with a temperature of 10^4 K. The third term is the expansion term due to Λ , and turns out to have a very small effect on the end behavior. The final term is the drag force, where $\dot{m}_{r>r_v}$ represents the fact that this term is only from the change in mass due to inelastic collisions with baryonic matter encountered when the shell leaves the halo.

The third equation (7) is the change in pressure, and arises from conservation of thermal energy of the hot interior plasma.

$$E_{th} = 2\pi r^3 p$$

$$\dot{E}_{th} = L - p \frac{dV}{dt}$$

The important consequence of this equation is the sensitivity of the system to the net luminosity.

$$L_{net} = L_{in} \Theta(t_{sf} - t) + L_{diss} - L_{comp} - L_{brem} - L_{ion} \quad (8)$$

The luminosity input to the system by the supernova is actually not as simple as E_{sn}/t_{sf} . As new material is added to the shell from the central galaxy, a certain amount of energy is required to bring the material up to the shell's current radius and velocity from assumed initialization at the galaxy radius. The result is

$$L_{in} = L_{sn} - \frac{1}{2} \dot{m}_{t < t_{sf}} \left(v^2 - \frac{2GM}{r} + \frac{2GM}{r_{init}} \right),$$

where $\dot{m}_{t < t_{sf}}$ represents that this is only the change in mass due to additions from the central galaxy.

The dissipation energy represents the fraction of kinetic energy lost during the inelastic accretion of matter that radiates back in the interior of the shell:

$$L_{diss} = \frac{1}{2} f_d m (v - u)^2$$

Of the three radiation terms, at high redshifts, energy lost via Compton scattering is significantly higher than that lost through bremsstrahlung radiation and ionization of surrounding material.

$$L_{comp} \propto \frac{(1+z)^4}{(1+z_{init})^{1.5}} r^3 p \quad (9)$$

The Compton luminosity is therefore dependent on the initial redshift when the shell forms and the thermal energy of the shell, as well as quickly decreasing as redshift decreases (and the temperature of the CMB decreases).

2.3 Energy Distribution

The final energy of the system will exceed the total amount of energy input by the supernova due to the initial kinetic energy of the material accreted outside of the halo because it is already moving with the velocity of the Hubble flow. In order to confirm that energy is conserved within the

system, it is necessary to factor in the energy contributions from the initial states of the accreted material as well as indirect energy loss not included in the radiated energy term.

The energy of the system at any point is the sum of its kinetic and thermal energy, gravitational potential energy, and energy lost from the system.

$$E = E_{kinetic} + E_{thermal} - U_{grav} + E_{lost} \quad (10)$$

Therefore, the change in energy of the system at any time is

$$\begin{aligned} \dot{E} = \dot{E}_k + \dot{E}_t - \dot{U}_g + \dot{E}_{lost} &= \frac{1}{2}\dot{m}_s v^2 + m v \dot{v} + 6\pi r^2 \dot{r} p \\ + 2\pi r^3 \dot{p} - G\left(\frac{\dot{M}_{enc} m_s}{r} + \frac{M_{enc} \dot{m}_s}{r} - \frac{M_{enc} m_s \dot{r}}{r^2}\right) &+ L_{rad} + L_{sc}, \end{aligned}$$

where L_{sc} is the shock cooling luminosity, the fraction of the dissipation energy that radiates away instead of into the shell. Using the differential equations that define the shell's motion (5), (6), and (7) this reduces to

$$\begin{aligned} \dot{E} = L_{net} + (L_{rad} + L_{sc}) + \frac{1}{2}\dot{m}_s v^2 - 4\pi r^2 v p_{ext} + \Omega_\Lambda(z) H(z)^2 m_s v r \\ - \dot{m}_{r>r_v} v(v-u) - G\left(\frac{\dot{M}_{enc} m_s}{r} + \frac{M_{enc} \dot{m}_s}{r}\right). \end{aligned}$$

To demonstrate the final distribution of the energy of the system it's best to then replace the net luminosity (8) with its components and equate this to (3). The final result is

$$\begin{aligned} L_{in} + \frac{1}{2}\dot{m}_{r>r_v} u^2 = \dot{E}_k + \dot{E}_t + L_{lost} + G\frac{M_{enc} m_s \dot{r}}{r^2} \\ + 4\pi r^2 v p_{ext} - \Omega_\Lambda(z) H(z)^2 m_s v r \end{aligned} \quad (11)$$

The end result is that the energy input by the supernova and the initial kinetic energy from the matter outside the halo, as well as a small portion from the Hubble expansion of the universe, is partly conserved in the kinetic, thermal, and gravitational potential of the new shell. The rest is lost through radiation and shock cooling, overcoming the external pressure of the infall gas, and through the energy required to shift new material from the galaxy, during the period of star formation, up to the shell's current position.

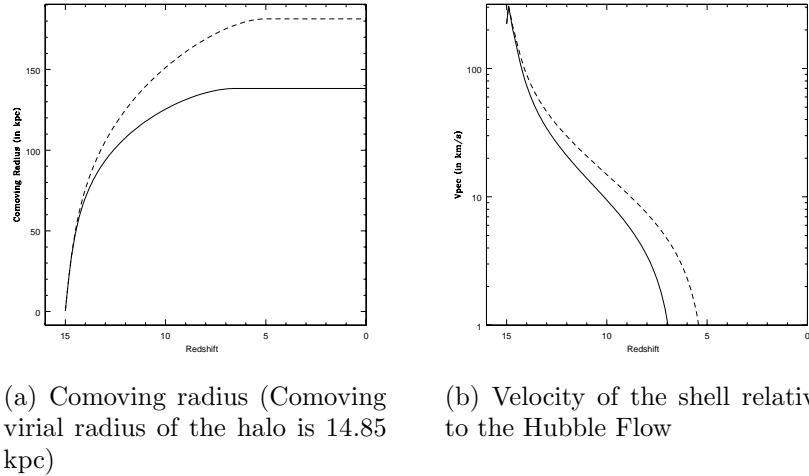


Figure 1: **Behavior of a shell with a total halo mass of $10^8 M_\odot$ and initial redshift of 15, comparing $f_d = 0.0$ (solid line) and $f_d = 1.0$ (dashed)**

3 Results

The end behavior of a standard shell is demonstrated in Figure 1. While star formation is still occurring, there is a driving positive luminosity within the system and it rapidly expands out past the halo. Once the star formation stops and the shell is colliding with increasing amounts of infall baryons, it decelerates. Eventually the velocity matches that of the Hubble flow, at which point the shell has reached its final size and is carried along with the rest of the Hubble flow until the present day.

The radius and velocity plots shown in Fig. 1 also demonstrate the effect of the dissipation parameter f_d . In the case where $f_d = 1.0$, the energy lost from the inelastic collisions contributes an additional source of positive luminosity and the shell can extend for a significantly longer period of time.

3.1 Asymptotic Behavior

The final size of the generated shells as a function of mass and redshift is shown in Figure 2. The mass range is bounded by the minimum mass for a baryonic over-density large enough to collapse into a galaxy capable of

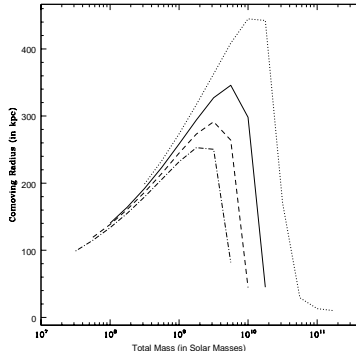


Figure 2: **Final radius of a shell vs. initial halo mass. Varied by initial redshift, starting with $z_0 = 20$ (dot-dashed), 15 (dashed), 10 (solid), and 5 (dot)**

undergoing star formation. This minimum mass is

$$M_{min} = \max(M_{cool}(z), M_{filter}(z)),$$

where M_{cool} is the mass required for temperature high enough for significant radiative cooling and M_{filter} is the modified Jean's mass limit.

At lower redshifts, the critical density of the universe decreases, which then leads to larger shells since they will accrete less matter and therefore lose less energy from collisions. However, in all cases there is a sharp asymptote past which if shells are capable of forming at all, they are significantly smaller, and constrained to within their halo. This is the case because as mass increases, total input supernova energy increases $\propto M$, but gravitational energy increases $\propto M^2$ since initial halo mass determines both the mass of the shell as well as the remaining mass of the dark matter halo and enclosed galaxy. Eventually there will be a point where initial net force is negative and the luminosity from the supernova isn't sufficient to keep the shell from being dragged into collapse by the galaxy.

Between the minimum bound on the mass and the asymptotic maximum, the final dependence of the radius on the redshift and initial halo mass can be approximated by simplifying the energy of the system to the input sources and the final kinetic energy, and assuming that the mass is dominated by

accreted matter from the surrounding matter.

$$E_{sn} + \frac{3}{10}f_d M_s(H(z_0)r)^2 = \frac{1}{2}M_s(H(z_0)r)^2$$

The f_d parameter represents whether or not the initial kinetic energy of the accreted matter can be included as an energy source, which is determined by whether the shell receives non-zero dissipation luminosity.

$$\begin{aligned} E_{sn} &= cM_s(H(z_0)r)^2 \\ L_{sn}t_{sf} &= c\left(\frac{4}{3}\pi r^3\rho_b(z_0)\right)(H(z_0)r)^2 \\ r &\approx 9.5238\left(\frac{M_t}{c(1+z_0)}\right)^{\frac{1}{5}}kpc \end{aligned}$$

Where M_t is the total initial mass of the halo and $c = 1/2$ if $f_d = 0$, and $1/5$ if $f_d = 1$. This bound is significantly larger than those shown in Fig. 2 ranging from several hundred kpc to a few Mpc. However, the slope is similar, suggesting that the proportionality,

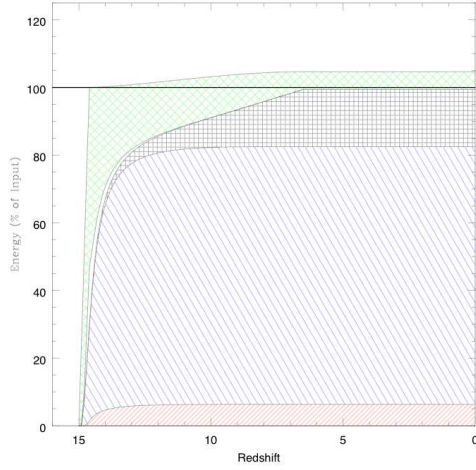
$$r \propto \left(\frac{M}{z}\right)^{\frac{1}{5}}$$

is an accurate description of the behavior. Using only four equally spaced data points on the graph, the exponent relating mass to final radius is between .22 and .23 for all four redshifts shown, close to the predicted value.

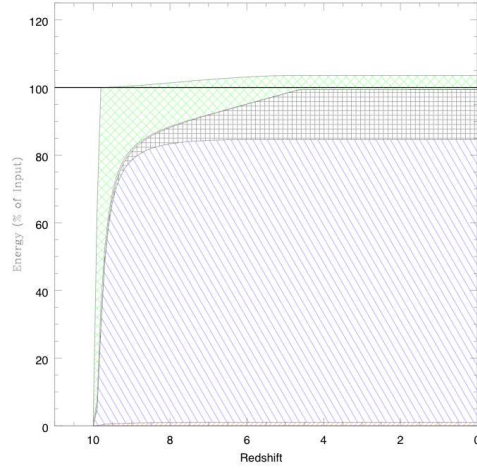
3.2 Parameter Dependence

In order to understand how the individual parameters affect the Compton energy contribution as well as the shell's behavior as a whole, several similar shell's were modeled with small changes in the parameters. The best way of understanding how their development changes is to look at the distribution of the system's energy over the shell's lifetime.

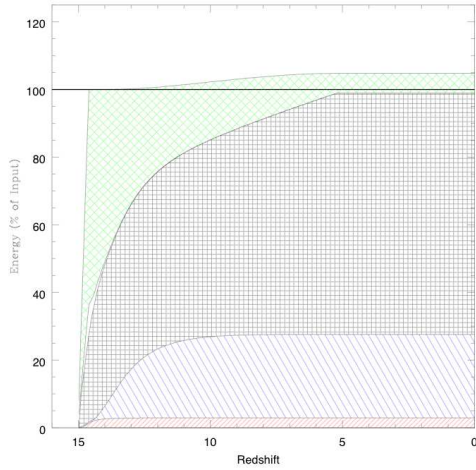
The graphs in Figure 3 show the distribution of the total energy of the shell, as a percent of the total input by the supernovae, differentiating between kinetic and thermal energy, work done by the force of gravity (referred to as gravitational energy), energy lost through Compton scattering, and all energy lost through any other form of radiation, including shock cooling.



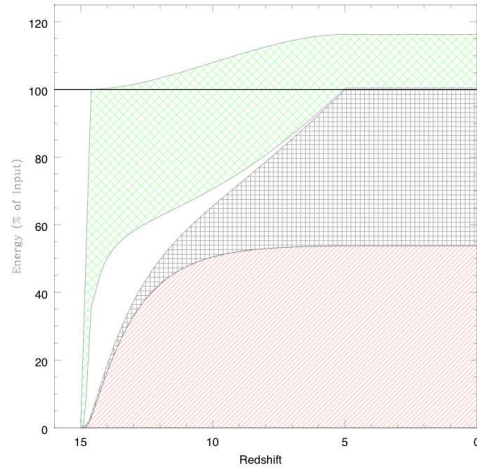
(a) Original shell



(b) Initial redshift, $z'_0 = 10$



(c) Initial mass, $M'_0 = 5 \cdot 10^9 M_\odot$



(d) Change in dissipation, $f_d = 1.0$

Figure 3: Comparison of energy distributions from shells with single parameter variations from one with initial mass $M_0 = 10^8 M_\odot$, initial redshift $z_0 = 15.0$, and $f_d = 0.0$. Sections are divided by where the energy ends up: Compton scattering (red slash), other radiation (blue slash), thermal (blank), work done by gravitational force (boxes), and kinetic energy (green diagonal boxes).

Comparing the original shell to those with modified initial redshift and mass, the behavior does not change drastically. In the case of a later initial redshift (*b*), the Compton luminosity dependence on initial redshift, as shown in (5), means that it contributes a smaller portion of the final energy. The contributions of other forms of radiation fill in this portion, and the shell expansion is approximately the same. In the case of larger initial mass (*c*), gravitational force grows $\propto M^2$ since it determines not only the shell mass, but the enclosed mass of the halo as well. Therefore gravitational energy is a much larger sink for energy in the system, and dominates the final distribution of the energy. In all these cases, thermal energy is only significant for a small portion of time, if at all, since it quickly drops once star formation ceases.

The case with significantly varied behavior is the case where $f_d = 1.0$ (*d*). First, this shows how large the shock cooling radiation is, especially compared to the other forms of radiation, bremsstrahlung and ionization, as the radiation term that doesn't include Compton energy is practically impossible to see on the graph. The added positive luminosity source of the dissipation energy ensures that thermal energy stays significant for a much longer period of time, and this in turn means that Compton scattering also continues for longer and makes up a larger final proportion of the energy of the system. Finally, the total final energy of the system is larger due to the fact that the shell extends further and therefore accretes substantially more material that already had an initial kinetic energy available to the system. This large shift in the amount of Compton scattering suggests that uncertainty in the value of parameter f_d will be one of the most significant contributors to the final calculation of the Compton y -parameter.

3.3 Effects of Compton Cooling

Though touched upon as part of the fractional distribution of energy in the previous section, in order to calculate the contributions of Compton cooling, the order of magnitude total energy lost through this manner is needed for an accurate estimation. The mass range over which contributions are significant are constricted by two different relationships - the asymptotic limits on halo formation, as shown in section 3.1, and the mass function of halos as a function of redshift. The latter is the number density of halos of a given mass by redshift, and for the purpose of this paper, the Sheth-Tormen calculation is used.

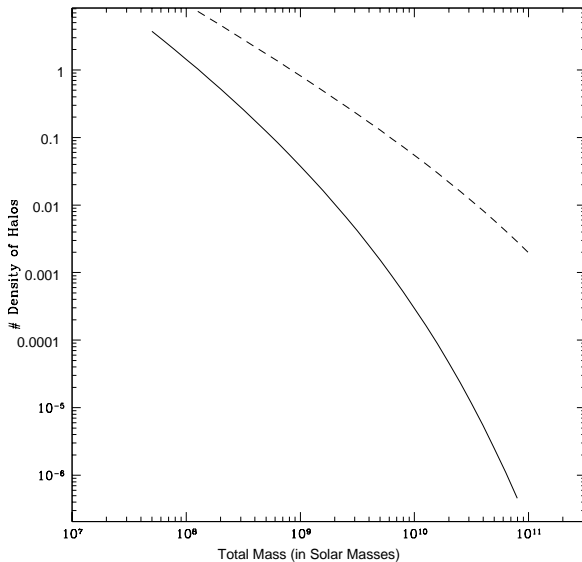


Figure 4: **Mass function (Sheth-Tormen) by mass at redshifts $z = 15$ (solid) and $z = 7$ (dashed)**

Graphing this density for redshifts within the expected range of the cosmic dawn, gives Figure 4. As shown in the graph, in both cases there is a fast decline in order of magnitude over the range of relevant masses constrained by the halo conditions. Extending this pattern further, we can then assume that even without the asymptote, if large halos could form and contribute large amounts of energy, the very low statistical likelihood of these shells existing constrains their final impact on the approximate value of the total Compton y -parameter to be negligible.

To determine the overall contributions from all halos, the amount of energy lost through Compton cooling must be summed weighted by the halo mass function.

$$E_C(z) = \int dM \frac{dn}{dM}(M, z) E_{Comp}(m, z) \quad (12)$$

The current results for the total Compton cooling by initial halo mass Figure 5 show that initial redshift does not have a significant effect on the total contribution in energy. The only change is a shift of the pattern over

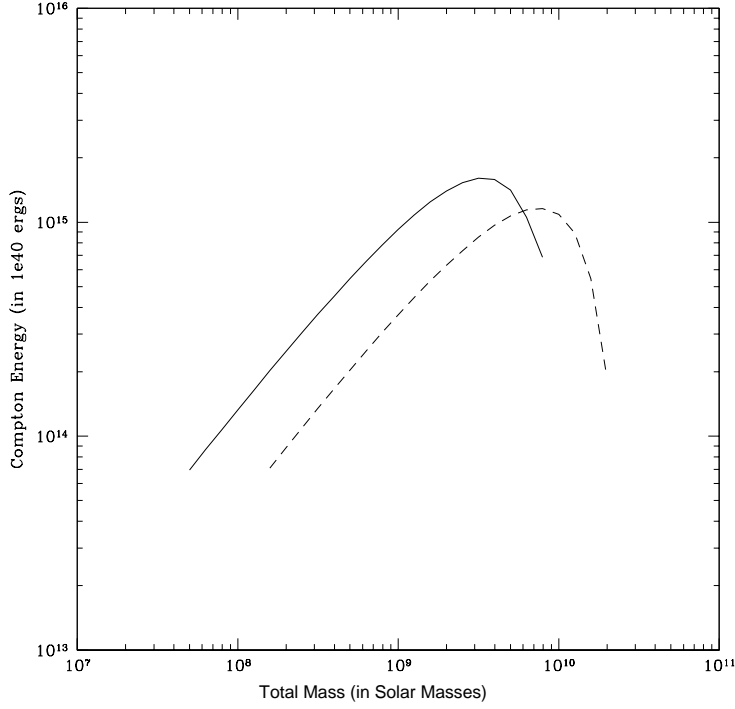


Figure 5: **Total energy lost via Compton cooling by initial halo mass at redshift $z = 15$ (solid) and $z = 7$ (dashed), both with $f_d = 1$**

to the right to account for the slight increase in the minimum and maximum bounding masses at latter redshift.

3.4 Sunyaev-Zeldovich Contribution

The final step to this research, currently in progress, is the final Compton y -parameter value that represents the total contributions of all supernovae shell contributions to the SZ effect. This value, assuming that the supernovae behave similarly and occur within a small range of redshift, can be approximated as,

$$y = -\frac{1}{2} \frac{\Delta T_\gamma}{T_\gamma} \approx -\frac{1}{8} \frac{\Delta U_\gamma}{U_\gamma} \approx 5 \cdot 10^{-6} \frac{10}{1 + z_{SN}} \frac{E_C/n_b}{100 \text{ eV}} \quad (13)$$

Where z_{sn} is the median redshift at which the supernovae are thought to occur, and E_C/n_b is the total amount of energy from Compton cooling per baryon in the universe. This second term is not directly calculated from the model, but can be solved by using f_C the fraction of energy from the supernovae that goes is lost through Compton cooling. Then this equation becomes,

$$y \approx 10^{-6} \frac{10}{1 + z_{SN}} \frac{f_C E_{SN}/n_b}{20 \text{ eV}}$$

This E_{SN}/n_b term is easier to solve for using parameters that are already defined by the model.

$$Q = \frac{N_{ion}}{\bar{n}_H v} = f_{star} f_{esc} f_{collapse} N_\gamma \quad (14)$$

This parameter Q is the fraction of the universe that is re-ionized, and can be used to relate parameters similar to those used in the shell model which are unknown, to known parameters $N_\gamma = 4000$, number of photons per baryon in stars, and $f_{esc} = 0.1$, photons that escape from the galaxy. To find the needed value E_{SN}/n_b , we start with calculating the energy generated by supernovae within some volume, V, of the universe.

$$\begin{aligned} \epsilon_{SN} V &= M_h a l o \frac{\Omega_b}{\Omega_0} \frac{10^{51} \text{ ergs}}{M_{SN}} f_{star} f_{collapse} \\ \epsilon_{SN} V &= \bar{\rho}_b V \frac{10^{51} \text{ ergs}}{M_{SN}} f_{star} f_{collapse} \\ \frac{\epsilon_{SN}}{\bar{n}_b} &= m_b \frac{10^{51} \text{ ergs}}{M_{SN}} f_{star} f_{collapse} \\ \frac{E_{SN}}{n_b} &= m_b \frac{10^{51} \text{ ergs}}{M_{SN}} \frac{Q}{N_\gamma f_{esc}} \end{aligned}$$

Assuming that the universe has been fully re-ionized, this solves to $E_{SN}/n_b = 10.4145 \text{ eV}$. Therefore, the only necessary component for finding y is the f_C parameter, which varies by mass, the final result is

$$y(z) = 10^{-6} \frac{10}{1 + z_{SN}} \frac{f_C E_{SN}/n_b}{20 \text{ eV}} \approx 4.33 \cdot 10^{-7} f_C \quad (15)$$

4 Discussion

The result of this paper thus far has mostly focused on the behavior of these matter shells as a function of the input parameters within the reasonable ranges relevant to the early universe. As shown previously, some fluctuations in parameters cause small shifts in behavior, such as in the case of a change in early redshift. However, parameters that have direct influence on the energy distribution of the system such as initial halo mass and especially the dissipation luminosity parameter f_d can cause large shifts in behavior that determine how long the shell is in a driven expansion mode and how quickly it reaches its asymptotic radius. Initial results suggest that the calculations of total Compton energy as a function of mass and redshift are correct. At constant redshift, values increase until the asymptotic behavior of the shell constricts the amount of expansion, therefore also decreasing the amount of Compton cooling. As redshift is modified, the relationship remains the same but shifts to match the expected changes in the relevant mass range. The final calculation of the y -parameter shown in (15), suggests that the result obeys the bound of $y < 1.5 \cdot 10^{-5}$ proposed by COBE FIRAS, but is at least an order of magnitude less than the calculation made by Oh, Cooray, & Kamionkowski, 2003. A more accurate calculation as a result of this model is still in progress.

References

- Barkana, R., & Loeb, A. 2001, Phys Rep., 349, 125
Fixsen D. J., Cheng E. S., Gales, J. M., Mather J. C., Shaffer R. A., & Wright E. L. 1996, ApJ, 473, 576
Furlanetto, S. R., & Loeb, A. 2001, ApJ, 556, 619
Furlanetto, S. R., & Loeb, A. 2003, ApJ, 588, 18
Oh, S. P., Cooray, A. & Kamionkowski, M. 2003, MNRAS, 342, L20
Ostriker, J. P., & McKee, C. 1988, Rev. Mod. Phys., 60, 1
Springel, V., & Hernquist, L. 2003, MNRAS, 339, 312
Tegmark, M., Silk, J., & Evrard, A. 1993, ApJ, 417, 54